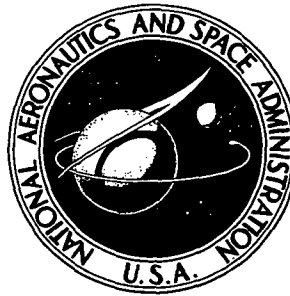


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AUTOMATED PRELIMINARY DESIGN OF
SIMPLIFIED WING STRUCTURES TO SATISFY
STRENGTH AND FLUTTER REQUIREMENTS

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SUMMARY

A simple structural model of an aircraft wing is used to show the effects of strength (stress) and flutter requirements on the design of minimum-weight aircraft-wing structures. The wing is idealized as an isotropic sandwich plate with a variable cover thickness distribution and a variable depth between covers. Plate theory is used for the structural analysis, and piston theory is used for the unsteady aerodynamics in the flutter analysis. Mathematical programming techniques are used to find the minimum-weight cover thickness distribution which satisfies flutter, strength, and minimum-gage constraints. This report presents the method of solution, some sample results, and the computer program used to obtain these results.

The results indicate that the cover thickness distribution obtained when designing for the strength requirement alone may be quite different from the cover thickness distribution obtained when designing for either the flutter requirement alone or for both the strength and flutter requirements concurrently. This conclusion emphasizes the need for designing for both flutter and strength from the outset.

INTRODUCTION

The speed and storage capacity of modern-day computers, together with the development of efficient computer-oriented decision-making strategies, have encouraged researchers to seek ways to use the computer for not only analysis purposes but also for portions of the design process. Research is underway in this area because automated design methods have the potential for providing superior designs in a shorter time and at a lower cost than is possible with more conventional design techniques. One of the objectives of automated design is to incorporate interdisciplinary design requirements into the design process. The present report considers one example of an interdisciplinary design problem – that of designing a wing structure to meet a strength (stress level) requirement and a flutter requirement. In this example, an optimization procedure is used to determine the lightest structure which will meet these requirements.

The purpose of this report is to show the effects of strength (stress) and flutter requirements on the design of simplified aircraft-wing structures. The wing is idealized as an isotropic sandwich plate with a variable cover thickness distribution and a variable depth between covers. Plate theory is used for the structural analysis, and piston theory is used for the unsteady aerodynamics in the flutter analysis. Mathematical programming techniques are used to find the minimum-weight cover thickness distribution which satisfies flutter, strength, and minimum-gage constraints. This report presents the method of solution, some sample results, and the computer program used to obtain these results.

Some previous work dealing with the design of aircraft structures to satisfy aeroelastic constraints may be found in references 1 to 5. Some studies of automated structural design for strength, buckling, dynamic and/or deflection constraints are contained in references 6 to 12.

SYMBOLS

The physical quantities used in this paper are given in both the International System of Units (SI) (ref. 13) and in the U.S. Customary System of Units. The measurements and calculations were made in U.S. Customary Units. Appendix A presents factors relating these two systems of units.

a	speed of sound
C	design variables
c	local chord length
D	local flexural stiffness of wing
E	modulus of elasticity
h	total local depth of wing
M	Mach number
m	mass of wing per unit area
N	number of spanwise stations on the wing (see fig. 3)
P	sum of the objective function and the penalty function (see eq. (16))
p	normal loading per unit area

r	penalty-function weighting factor (see eq. (16))
t	local thickness of wing covers
v	velocity
v_f	flutter velocity
W, w	assumed normal deflection of wing neutral surface (see eqs. (1) and (2))
X, Y, Z	chordwise, spanwise, and normal axes, respectively
x, y	chordwise and spanwise coordinates, respectively
x_L	value of x at wing leading edge
x_T	value of x at wing trailing edge (zero for all calculations presented herein)
y_S	value of y at wing tip, semispan
α	real part of flutter determinant
β	imaginary part of flutter determinant
γ	ratio of specific heats
ϵ	spanwise finite-difference increment
$\epsilon_x, \epsilon_y, \gamma_{xy}$	components of normal and shearing strain in wing cover plates
μ	Poisson's ratio
ρ	mass density of air
σ_{cr}	critical value of the stress defined by the Von Mises stress condition
$\sigma_x, \sigma_y, \tau_{xy}$	components of normal and shearing stress in wing cover plates
τ	time

φ_i	deflection coefficients (see eq. (2))
ω	frequency, rad/s
ω_f	flutter frequency

Dots indicate differentiation with respect to time.

ANALYSIS

The analysis procedures used in this report are plate theory for the wing structural analysis, piston theory for the unsteady aerodynamics in the flutter analysis, and mathematical programming techniques for finding the minimum-weight design. These three procedures are discussed briefly in the next three sections. Additional discussion and detailed definitions of matrix expressions are given in appendix B.

Plate Theory for Wing Structural Analysis

The wing is assumed to be sufficiently thin to be analyzed with plate theory. It is further assumed that internal bulkheads, ribs, and spars exist which prevent transverse shear deflections, but any other load-carrying capacity of these structural elements is neglected. All load is assumed to be carried by a set of wing cover plates – an upper cover plate and a lower cover plate. The wing structural analysis is concerned only with these cover plates. The wing is assumed to have a symmetric biconvex airfoil shape and, therefore, the distance between covers varies over the surface of the wing (fig. 1). The covers have a variable thickness distribution which is adjusted in the design procedure to provide the minimum-weight design.

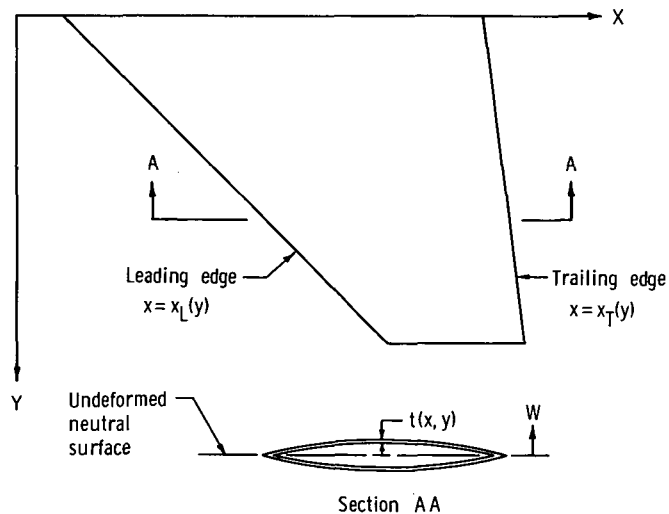


Figure 1.- Wing coordinate system.

The wing structural analysis is based upon the approach presented in reference 14. The normal deflection W of the wing neutral surface is assumed to be given by

$$W(x,y,\tau) = w(x,y)e^{i\omega\tau} \quad (1)$$

where

$$w(x,y) = \varphi_0(y) + x\varphi_1(y) + x^2\varphi_2(y) \quad (2)$$

The assumed solution includes terms only to the second degree in the x (chordwise) direction. Finite differences in the y (spanwise) direction are used to solve for the unknowns $\varphi_0(y)$, $\varphi_1(y)$, and $\varphi_2(y)$. The wing coordinate system and deflection shape are shown in figures 1 and 2, respectively. The location of the Y -axis is arbitrary.

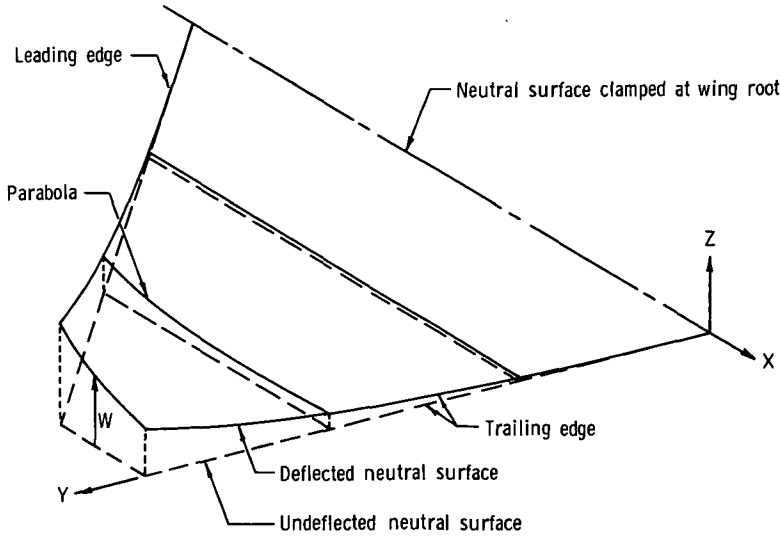


Figure 2.- Diagram of undeflected and deflected neutral surface of wing showing assumed parabolic chordwise-deflection shape.

An energy approach (ref. 14) is used to obtain the equilibrium equations for a variable-thickness isotropic plate or for an isotropic built-up plate with variable depth. The matrix form of these equilibrium equations is

$$\begin{bmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \varphi_0 \\ \varphi_1 \\ \varphi_2 \end{bmatrix} = \epsilon^4 \begin{bmatrix} p_0 \\ p_1 \\ p_2 \end{bmatrix} \quad (3)$$

in which the matrix $[A_{ij}]$ is composed of terms involving the stiffness and derivatives of the stiffness, the vector $[\phi_i]$ describes the deflections, the vector $[p_i]$ is the normal loading, and ϵ is the spanwise finite difference increment shown in figure 3. The matrix expressions are presented in detail in appendix B. The normal loading used in the strength analysis is a uniform normal loading. The wing is assumed to be fixed at the root.

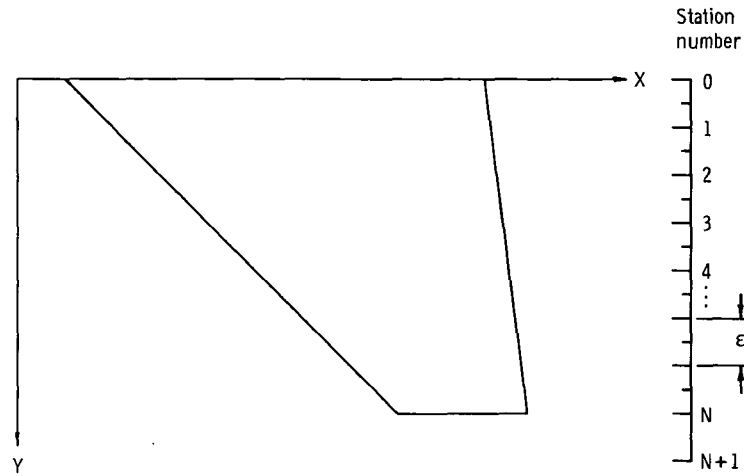


Figure 3.- Location of stations on planform of half-wing.

Once the deflections have been calculated from equations (3) and (2), the strains in the wing covers are calculated by using the following strain-displacement relations:

$$\epsilon_x = \mp \frac{h}{2} \frac{\partial^2 W}{\partial x^2} \quad (4)$$

$$\epsilon_y = \mp \frac{h}{2} \frac{\partial^2 W}{\partial y^2} \quad (5)$$

$$\gamma_{xy} = \mp h \frac{\partial^2 W}{\partial x \partial y} \quad (6)$$

where the upper and lower signs are for the upper and lower wing covers, respectively, and h is the local distance between the upper wing-cover midplane and the lower wing-cover midplane. The stresses can then be calculated by using the following stress-strain relations:

$$\sigma_x = \frac{E}{1 - \mu^2} (\epsilon_x + \mu \epsilon_y) \quad (7)$$

$$\sigma_y = \frac{E}{1 - \mu^2} (\epsilon_y + \mu \epsilon_x) \quad (8)$$

$$\tau_{xy} = \frac{E}{2(1 + \mu)} \gamma_{xy} \quad (9)$$

These stresses are used in the strength constraint discussed in a subsequent section entitled "Mathematical Programming Techniques for Determining Minimum-Weight Design."

Piston Theory for Unsteady Aerodynamics in Flutter Analysis

In contrast to the uniform normal loading used in the strength analysis of the wing, the normal loading used in the flutter analysis is a function of x , y , and τ (time). The total loading that acts on an airfoil in motion is the sum of the inertia and unsteady aerodynamic loadings. (Structural damping is neglected.) Based on second-order piston theory for symmetrical airfoils (refs. 15 and 16), the sum of these two loadings is

$$p(x,y,\tau) = -m\ddot{W} - 2\rho a \left(1 + \frac{\gamma + 1}{4} M \frac{\partial h}{\partial x} \right) \left(\frac{\partial}{\partial \tau} + v \frac{\partial}{\partial x} \right) W \quad (10)$$

The unsteady loading $p(x,y,\tau)$ can be taken to the left side of the equilibrium equation (eq. (3)), thereby allowing the equation to be written in the form

$$\left\{ \begin{bmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{bmatrix} + \epsilon^4 \begin{bmatrix} B_{00} & B_{01} & B_{02} \\ B_{10} & B_{11} & B_{12} \\ B_{20} & B_{21} & B_{22} \end{bmatrix} \right\} \begin{bmatrix} \varphi_0 \\ \varphi_1 \\ \varphi_2 \end{bmatrix} = 0 \quad (11)$$

Detailed expressions which define the components of equation (11) are given in appendix B.

The flutter speed v_f and the flutter frequency ω_f are calculated by setting the complex determinant of the matrix of coefficients of φ_i in equation (11) equal to zero — that is,

$$\left| \begin{bmatrix} A_{ij} + \epsilon^4 B_{ij} \end{bmatrix} \right| = \alpha + i\beta = 0 \quad (12)$$

There are, therefore, two equations

$$\alpha = 0 \quad (13)$$

$$\beta = 0 \quad (14)$$

in the two unknowns v_f and ω_f . First, the solution neighborhood for equation (12) is established by evaluating the determinant for many combinations of ω and v . The Newton-Raphson method is then used to determine ω_f and v_f precisely. An example of this solution technique is given in appendix C.

Mathematical Programming Techniques for

Determining Minimum-Weight Design

The thickness distribution of the wing covers is adjusted to give the minimum-weight design which satisfies all the design requirements. The thickness distribution is assumed to be of the form

$$t(x,y) = \sum_i \sum_j C_{ij} x^i y^j \quad (15)$$

The coefficients C_{ij} are design variables which are adjusted to give the minimum-weight design. Henceforth, the design variables will be referred to as the vector of design variables denoted by \vec{C} . The detailed expressions used for $t(x,y)$ are given in a subsequent section entitled "Applications and Results."

In this report the constraints, which are the design requirements, are introduced into the problem by way of an interior penalty function. This method of treating constraints is sometimes called "SUMT" – Sequential Unconstrained Minimization Technique (refs. 17 and 18). In SUMT, the objective function, which is taken to be the weight in this study, is augmented by a penalty term that accounts for the constraints. The combination of these two terms is called the P function:

$$P(\vec{C}, r) = \text{Weight}(\vec{C}) + r \sum_j \frac{1}{g_j(\vec{C})} \quad (16)$$

The term $r \sum_j \frac{1}{g_j(\vec{C})}$ is the penalty term in which r is a weighting factor and $g_j(\vec{C})$

are the inequality constraints. The inequality constraints are formulated in such a way that they may be written as

$$g_j(\vec{C}) > 0 \quad (17)$$

The penalty term repels the P function from the inequality constraints and transforms the problem from a constrained minimization problem to an unconstrained problem. By carrying out a sequence of minimizations of the P function corresponding to a sequence of reductions in the value of the weighting factor r , it is possible to obtain the vector of design variables which satisfies the constraints and gives the minimum value of the objective function. The constraints used herein are as follows:

(1) For flutter, the flutter speed v_f shall not be less than some prescribed critical flutter speed $v_{f,cr}$. This constraint can be expressed mathematically as

$$g_1 \equiv 1 - \frac{v_{f,cr}}{v_f} > 0 \quad (18)$$

(2) For strength, at no point on the wing covers shall the stress exceed the prescribed critical stress, or

$$g_2 \equiv \frac{1}{\frac{1}{A} \iint_A \frac{dA}{1 - \frac{\sigma}{\sigma_{cr}}}} > 0 \quad (19)$$

where A is the wing area,

$$\sigma = \left(\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2 \right)^{1/2}$$

which is the Von Mises stress condition, and σ_{cr} is the critical value of σ .

(3) For minimum gage, at no point on the wing covers shall the thickness t be less than some prescribed minimum thickness t_{min} , or

$$g_3 \equiv \frac{1}{\frac{1}{A} \iint_A \frac{dA}{1 - \frac{t_{min}}{t}}} > 0 \quad (20)$$

Once a set of design variables has been defined (eq. (15)), and the design requirements have been formulated as constraints (eqs. (18) to (20)), the wing can be designed by using mathematical programming methods. Additional comments on the math programming approach used are given in appendix B.

A computer program denoted "SWIFT" which incorporates the structural analysis, flutter analysis, and optimization techniques just discussed is presented in appendix C.

In order to check the validity of the structural analysis, natural-vibration frequencies for a trapezoidal flat plate were calculated by using the preceding analysis and are compared with experimentally determined natural frequencies in appendix D. A theoretical flutter speed and frequency for that plate are also presented in appendix D but are not compared with experimental results.

APPLICATIONS AND RESULTS

In order to show the effects of flutter and strength requirements on the design of aircraft wings, these requirements were used in the calculation of several minimum-weight wing cover thickness distributions. In these sample calculations, the number and choice of design variables is varied and the design requirements are varied. The wing properties used in all calculations are presented in figure 4. For the examples presented herein, the Y-axis is chosen to lie along the wing trailing edge. The number of spanwise stations N used in the structural analysis is six.

The following specific examples were considered:

First, the following set of design requirements was chosen:

(1) For flutter, the wing is to be flutter-free at a dynamic pressure of 160 kN/m^2 (3340 lbf/ft^2) at an altitude of 7620 meters (25 000 ft). (This corresponds to $M = 2.46$.)

(2) For strength, the wing is to support a uniformly distributed loading of 6.89 kN/m^2 (144 lbf/ft^2).

(3) The minimum-gage requirement is 0.0508 centimeter (0.0200 in.).

These design requirements, together with a sixteen-design-variable polynomial, were used to obtain a flutter design (requirements 1 and 3), a strength design (requirements 2 and 3), and a design (denoted as a "strength-flutter design" or "combined design") which meets the flutter, strength, and minimum-gage requirements.

Second, these same design requirements, together with a six-design-variable polynomial, were also used to obtain a flutter design, a strength design, and a strength-flutter design.

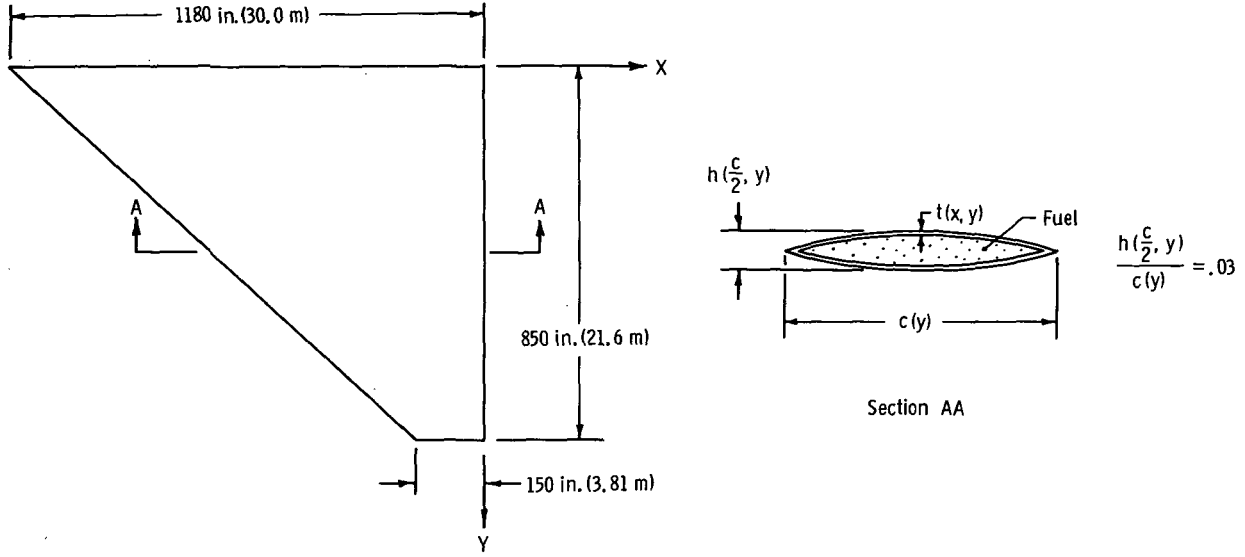


Figure 4.- Wing description and cover-material properties used in the sample calculations. Fuel mass, 150 000 lbm (68.0 Mg). Properties of titanium material: $E = 16\,400$ ksi (113 GN/m²); $\mu = 0.3$; $\sigma_{cr} = 125$ ksi (863 MN/m²); density, 0.160 lbm/in³ (4430 kg/m³).

Third, the flutter and strength design requirements were varied and the mass of minimum-weight wings designed by using the six-design-variable polynomial was calculated as a function of the design requirements.

Results Obtained With Sixteen Design Variables

The thickness distribution chosen for the first example is a sixteen-term polynomial. That polynomial is

$$\begin{aligned}
 t = & C_1 + C_2 \left(1 - \frac{y}{y_S}\right) + C_3 \left(1 - \frac{y}{y_S}\right)^2 + C_4 \left(1 - \frac{y}{y_S}\right)^3 \\
 & + \left[C_5 + C_6 \left(1 - \frac{y}{y_S}\right) + C_7 \left(1 - \frac{y}{y_S}\right)^2 + C_8 \left(1 - \frac{y}{y_S}\right)^3 \right] \left(1 - \frac{2x}{x_L}\right) \\
 & + \left[C_9 + C_{10} \left(1 - \frac{y}{y_S}\right) + C_{11} \left(1 - \frac{y}{y_S}\right)^2 + C_{12} \left(1 - \frac{y}{y_S}\right)^3 \right] \left(1 - \frac{2x}{x_L}\right)^2 \\
 & + \left[C_{13} + C_{14} \left(1 - \frac{y}{y_S}\right) + C_{15} \left(1 - \frac{y}{y_S}\right)^2 + C_{16} \left(1 - \frac{y}{y_S}\right)^3 \right] \left(1 - \frac{2x}{x_L}\right)^3
 \end{aligned} \tag{21}$$

in which x is the chordwise coordinate, y is the spanwise coordinate, x_L is the value of x at the wing leading edge, y_S is the semispan, and C_i ($i = 1, 2, 3, \dots, 16$) are the design variables. For clarity, the spanwise and chordwise variations corresponding to each term in equation (21) are shown in figure 5. The thickness distribution for the upper cover is the same as for the lower cover.

$$\begin{aligned}
 t = & \left[\begin{array}{c} R \quad T \\ c_1 \text{ [rectangle]} + c_2 \text{ [triangle]} + c_3 \text{ [triangle]} + c_4 \text{ [triangle]} \end{array} \right] \times \left[\begin{array}{c} x_L \quad x_T \\ \text{[rectangle]} \end{array} \right] \\
 & + \left[\begin{array}{c} c_5 \text{ [rectangle]} + c_6 \text{ [triangle]} + c_7 \text{ [triangle]} + c_8 \text{ [triangle]} \end{array} \right] \times \left[\begin{array}{c} \text{[trapezoid]} \end{array} \right] \\
 & + \left[\begin{array}{c} c_9 \text{ [rectangle]} + c_{10} \text{ [triangle]} + c_{11} \text{ [triangle]} + c_{12} \text{ [triangle]} \end{array} \right] \times \left[\begin{array}{c} \text{[curved shape]} \end{array} \right] \\
 & + \left[\begin{array}{c} c_{13} \text{ [rectangle]} + c_{14} \text{ [triangle]} + c_{15} \text{ [triangle]} + c_{16} \text{ [triangle]} \end{array} \right] \times \left[\begin{array}{c} \text{[trapezoid]} \end{array} \right]
 \end{aligned}$$

Figure 5.- Spanwise and chordwise variations for each term in equation (21).
 R = root; T = tip; x_L = leading edge; x_T = trailing edge.

Contour plots showing wing cover thickness distributions for the flutter, strength, and combined designs are shown in figure 6. These plots show contours for a single cover. If only the flutter and minimum-gage requirements are considered, the lightest cover thickness distribution that can be obtained from the sixteen design variables described in equation (21) is shown in figure 6(a). The mass of the semispan wing (that is, the mass of both the top and the bottom cover) is 3160 kilograms (6960 lbm). This flutter design will support a uniform static pressure loading of only 2.82 kN/m² (58.9 lbf/ft²). If only the strength and minimum-gage requirements are considered, the lightest cover thickness distribution for the sixteen prescribed design variables is shown in figure 6(b). The cover mass is 3220 kilograms (7100 lbm). This strength design will flutter at a dynamic pressure of 51.2 kN/m² (1070 lbf/ft²). The cover thickness distribution for the flutter design is quite different from that for the strength design. The design which satisfies both the flutter and strength requirement is shown in figure 6(c). The cover mass for this design is 4030 kilograms (8890 lbm).

Because of the large mass of fuel (68.0 megagrams (150 000 lbm)) carried in the wing, the contribution of the wing covers to the total mass distribution is small but is, nevertheless, included in the flutter analysis used to obtain these results. The primary effect of the thickness distribution is to establish the stiffness distribution.

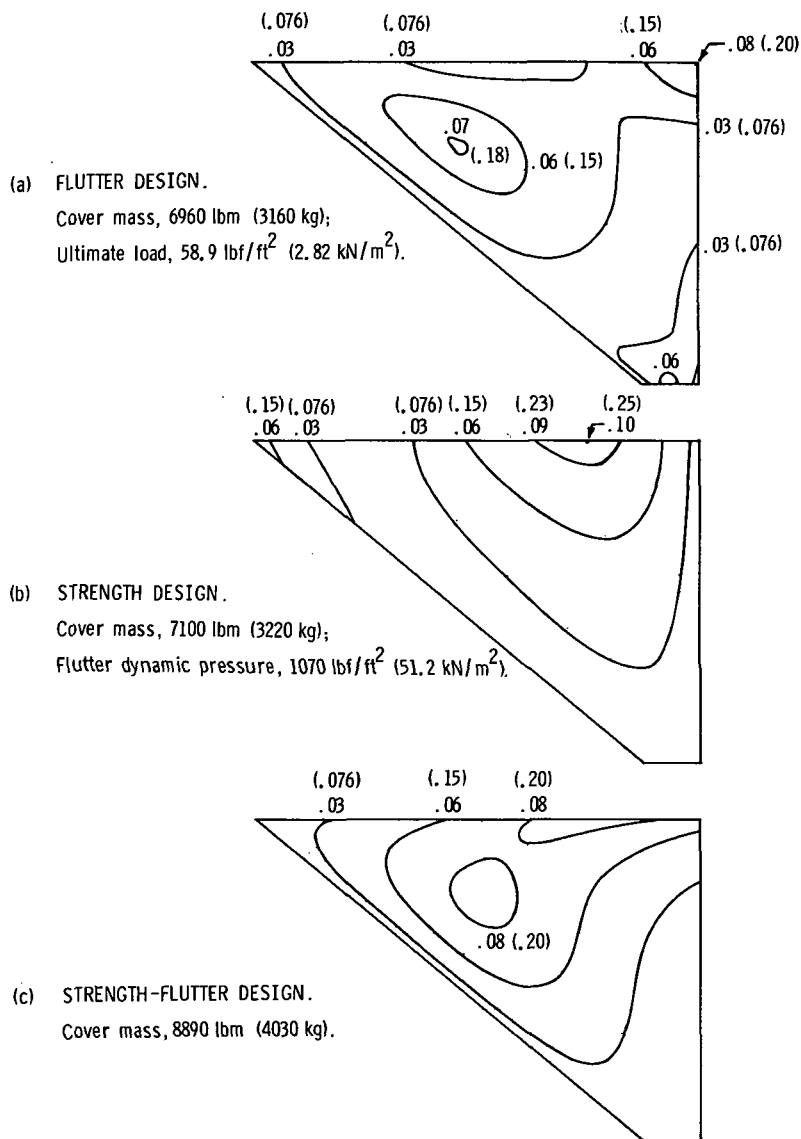


Figure 6.- Contour plots showing wing cover thickness distributions for flutter, strength, and combined designs obtained by using sixteen design variables. Contour labels are in inches (centimeters).

Results Obtained With Six Design Variables

Calculations were also made with a judicious combination of certain terms in equation (21). The thickness expression which was used is

$$t = t_{\min} + \left[C_1 \left(1 - \frac{y}{y_S} \right) + C_2 \left(1 - \frac{y}{y_S} \right)^2 + C_3 \left(1 - \frac{y}{y_S} \right)^3 \right] \left[1 - \left(1 - \frac{2x}{x_L} \right)^2 \right] + \left[C_4 \left(1 - \frac{y}{y_S} \right) + C_5 \left(1 - \frac{y}{y_S} \right)^2 + C_6 \left(1 - \frac{y}{y_S} \right)^3 \right] \left[1 - \left(1 - \frac{2x}{x_L} \right)^2 \right] \left(1 - \frac{2x}{x_L} \right) \quad (22)$$

in which $t_{\min} = 0.0508$ centimeter (0.0200 in.), the minimum-gage requirement. The spanwise and chordwise variations for each term in equation (22) are shown in figure 7. The thickness distribution described by equation (22) is minimum gage along the leading and trailing edges and at the tip – a reasonable limitation.

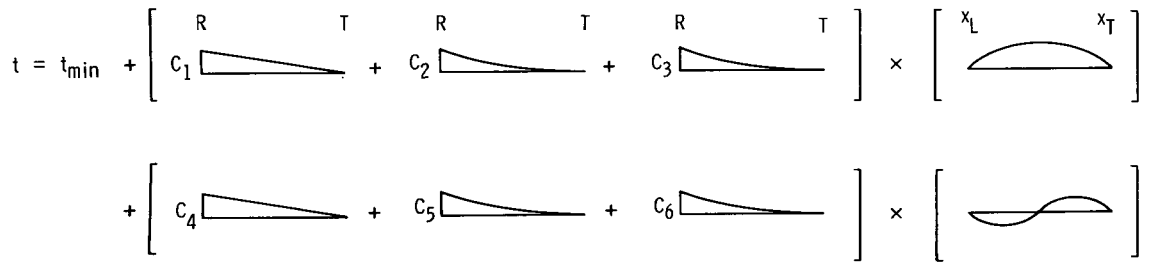


Figure 7.- Spanwise and chordwise variations for each term in equation (22).
R = root; T = tip; x_L = leading edge; x_T = trailing edge.

All of the thickness variations described by equation (22) can also be obtained from equation (21). Therefore, the design obtained by using equation (22) cannot be lighter than the design obtained by using equation (21). Equation (22), however, contains only six design variables, compared with sixteen design variables in equation (21). Since computer time for the design process depends strongly upon the number of design variables, it is advantageous to use a small number of well-chosen design variables rather than a large number of arbitrarily chosen design variables.

The design requirements used for this second example are the same as those previously stated. Contour plots showing wing cover thickness distributions for the flutter, strength, and combined designs are shown in figures 8(a), 8(b), and 8(c), respectively. As in the preceding example, the cover thickness distribution for the flutter design is quite different from that for the strength design.

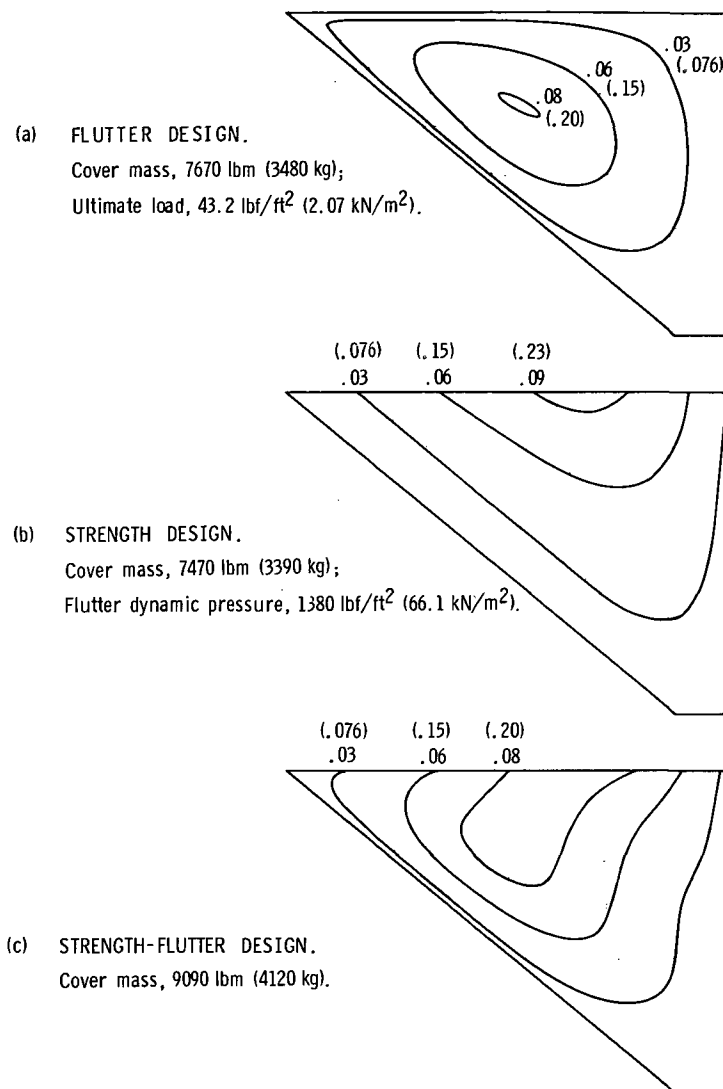


Figure 8.- Contour plots showing wing cover thickness distributions for flutter, strength, and combined designs obtained by using six design variables. Contour labels are in inches (centimeters).

For the flutter design, the new six-design-variable thickness distribution is 10 percent heavier than the sixteen-design-variable thickness distribution. The new strength design is only 5 percent heavier than the sixteen-design-variable case. The new combined design is only 2 percent heavier than the sixteen-design-variable case.

Results Obtained for Various Strength and Flutter Design Requirements

The wing properties given in figure 4 were used in calculating several other minimum-weight thickness distributions for various strength and flutter design requirements. All of these calculations were made with the six-design-variable polynomial

given in equation (22). The results are presented in figure 9 in which minimum-weight wing cover mass is shown as a function of flutter (abscissa) and strength (ordinate) design requirements. Lines of constant cover mass are shown as dashed lines. The solid curve which is almost horizontal represents designs dictated by flutter requirements, and the solid curve that is almost vertical represents designs dictated by strength requirements. Points a, b, and c indicate the flutter, strength, and combined designs shown in figures 8(a), 8(b), and 8(c), respectively.

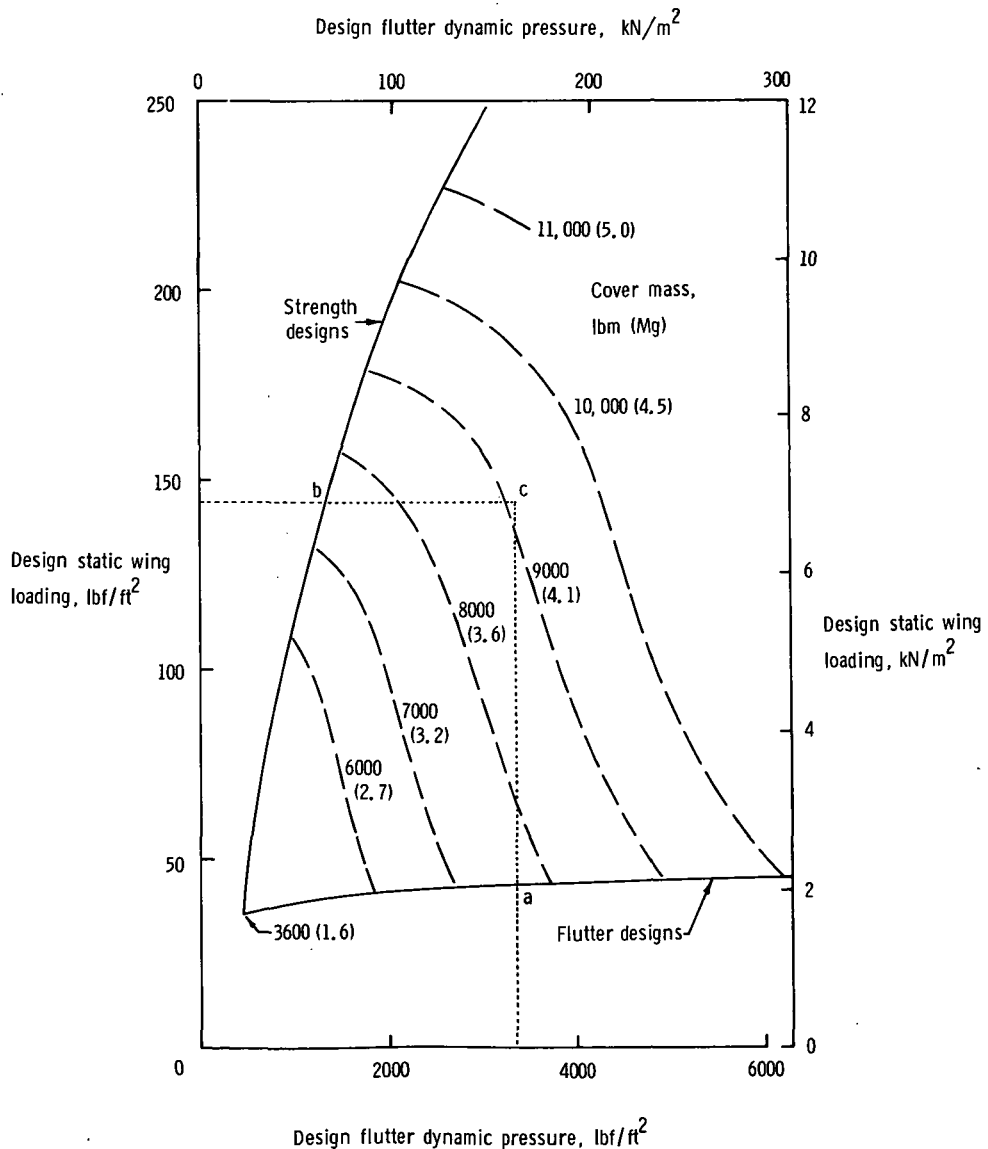


Figure 9.- Minimum-weight wing cover mass as a function of design flutter dynamic pressure (flutter requirement) and design static wing loading (strength requirement).

The results presented in figure 9 indicate that for this example there is little interaction between strength and flutter design requirements. For a strength-designed wing, improving the strength in the best possible way improves the flutter characteristics only slightly. For a flutter-designed wing, increasing the flutter speed in the best possible way has a negligible effect on the strength. The fact that designing for strength is quite different from designing for flutter emphasizes the need for designing for both strength and flutter from the outset.

There are many ways to obtain a design which satisfies both the flutter and strength design requirements given by point c in figure 9. One approach is to start with the strength design shown at point b and increase the thickness uniformly over the entire wing until the flutter requirement is satisfied. For the present set of design conditions, the resulting design is 38 percent heavier than the minimum-weight combined design at point c.

A second approach is to use the envelope of the thickness distributions for the flutter and strength cases. That is, at each point on the wing the cover thickness is taken to be the larger of the two following thicknesses: (1) the thickness at that point for the flutter design, and (2) the thickness at that point for the strength design. In the present case, the resulting envelope design does meet the design requirements and is only about 3 percent heavier than the combined design at point c. However, all envelope designs do not meet the design requirements. For example, if the strength design requirement given by point c in figure 9 is reduced 50 percent to 3.45 kN/m^2 (72 lbf/ft^2) without changing the flutter design requirement, the envelope design corresponding to the new set of design requirements is about 2 percent lighter than the minimum-weight combined design for that same set of design requirements. However, the envelope design flutters at a dynamic pressure about 9 percent lower than the flutter design requirement.

A third possible approach is to use the strength design as a minimum-gage requirement and, by using an optimization technique, add material to meet the flutter requirement. Such a design would be lighter than an acceptable envelope design. There are, of course, many other approaches.

The intent of this report is not to prove that a combined design approach as characterized by the math programming techniques used herein is superior to any other design approach. There are, however, two items to consider with respect to the combined design approach. First, for the set of design variables considered, the combined design approach will give the lightest design that meets all design requirements. In this simple model which combines two disciplines – structural analysis and aeroelasticity – the weight penalty caused by designing separately and sequentially rather than concurrently may be small. However, as the model becomes more complex and other disciplines are added, that penalty may increase. Second, for the optimization techniques described herein it is

faster to carry out a combined design than to design for strength and then to design for flutter. It is realized, however, that there are specialized design techniques that are tailored for a particular discipline. These techniques may be well-suited for sequential design and may provide an adequate design with shorter computer run times. The effectiveness of these design techniques can be assessed by comparing the results obtained by using these techniques with the results obtained by using a combined-design mathematical programming approach.

CONCLUDING REMARKS

A simple structural model of an aircraft wing has been used to show the effects of strength and flutter requirements on the design of minimum-weight wing structures. The wing structural analysis is based on plate theory. Piston theory is used in the flutter analysis. This report has presented the method of solution, some sample results, and the computer program used to obtain these results.

Wings were designed by adjusting the coefficients of polynomials which describe the wing cover thickness distribution. Sample calculations were carried out for a polynomial containing six coefficients (design variables) and for a polynomial containing sixteen coefficients. Mathematical programming techniques were used to determine the values of the coefficients which gave the lightest cover thickness distribution that satisfied flutter, strength, and minimum-gage requirements.

The results indicate that the cover thickness distribution obtained when designing for the strength requirement alone may be quite different from the cover thickness distribution obtained when designing for either the flutter requirement alone or for both the strength and flutter requirements concurrently. This conclusion emphasizes the need for designing for both flutter and strength from the outset.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., October 27, 1971.

APPENDIX A

CONVERSION OF U.S. CUSTOMARY UNITS TO SI UNITS

The International System of Units (SI) was adopted by the Eleventh General Conference on Weights and Measures, Paris, October 1960 (ref. 13). Conversion factors for the units used herein are given in the following table:

Physical quantity	U.S. Customary Unit	Conversion factor (*)	SI Unit (**)
Force	lbf	4.44822	newtons (N)
Length	{ ft in.	{ 0.3048 0.0254	{ meters (m)
Mass	lbm	0.45359	kilograms (kg)
Stress, modulus . . .	ksi	6.895×10^6	newtons/meter ² (N/m ²)
Density	lbm/in ³	27.68×10^6	grams/meter ³ (g/m ³)
Pressure	lbf/ft ²	47.88	newtons/meter ² (N/m ²)
Speed	ft/s	0.3048	meters/second (m/s)

*Multiply value given in U.S. Customary Units by conversion factor to obtain equivalent value in SI Units.

**Prefixes to indicate multiples of units are as follows:

Prefix	Multiple
giga (G)	10 ⁹
mega (M)	10 ⁶
kilo (k)	10 ³
deci (d)	10 ⁻¹
centi (c)	10 ⁻²
milli (m)	10 ⁻³

APPENDIX B

DEFINITIONS OF MATRIX EXPRESSIONS AND FURTHER DISCUSSION OF ANALYSIS

Plate Theory for Wing Structural Analysis

For convenience, the plate equilibrium equation (eq. (3)) is repeated here

$$\begin{bmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \varphi_0 \\ \varphi_1 \\ \varphi_2 \end{bmatrix} = \epsilon^4 \begin{bmatrix} p_0 \\ p_1 \\ p_2 \end{bmatrix} \quad (B1)$$

The matrix components of equation (B1) are

$$\left. \begin{aligned} [A_{00}] &= [D_0] [a_0] [D_0]' \\ [A_{01}] &= [D_0] [a_1] [D_0]' \\ [A_{02}] &= [D_0] \left([a_2] [D_0]' + 2\mu\epsilon^2 [\bar{a}_0] \right) \\ [A_{11}] &= [D_0] [a_2] [D_0]' + 2(1 - \mu)\epsilon^2 [D_3] [a_0^*] [D_3]' \\ [A_{12}] &= [D_0] [a_3] [D_0]' + 4(1 - \mu)\epsilon^2 [D_3] [a_1^*] [D_3]' + 2\mu\epsilon^2 [D_0] [\bar{a}_1] \\ [A_{22}] &= [D_0] [a_4] [D_0]' + 8(1 - \mu)\epsilon^2 [D_3] [a_2^*] [D_3]' + 2\mu\epsilon^2 \left\{ [D_0] [\bar{a}_2] \right. \\ &\quad \left. + \left([D_0] [\bar{a}_2] \right)' \right\} + 4\epsilon^4 [\hat{a}_0] \\ [A_{10}] &= [A_{01}]' \quad [A_{20}] = [A_{02}]' \quad [A_{21}] = [A_{12}]' \end{aligned} \right\} \quad (B2)$$

Primes indicate the transpose of a matrix, and ϵ is the finite-difference increment in the spanwise direction. The finite-difference increments are shown in figure 3.

APPENDIX B – Continued

In this report the wing is assumed to be fixed at the root. Because more general boundary conditions are considered in reference 14, some of the expressions used herein differ slightly from the comparable expressions in reference 14. For the most part, however, the quantities defined herein are the same as those defined in reference 14. The matrices in equations (B2) are defined as follows (where the number of rows and columns, in that order, are indicated below the equation numbers):

$$[D_0] = \begin{bmatrix} 2 & -2 & 1 & & & \\ & 1 & -2 & 1 & & \\ & & 1 & -2 & 1 & \\ & & & \cdot & \cdot & \cdot \\ & & & & 1 & -2 & 1 \\ & & & & & 1 & -2 \\ & & & & & & 1 \end{bmatrix} \quad \begin{array}{l} \text{(B3)} \\ (N+1) \times (N+1) \end{array}$$

$$[D_3] = \begin{bmatrix} 1 & -1 & & & & \\ & 1 & -1 & & & \\ & & 1 & -1 & & \\ & & & \cdot & \cdot & \cdot \\ & & & & 1 & -1 \\ & & & & & 1 \\ & & & & & & 0 \end{bmatrix} \quad \begin{array}{l} \text{(B4)} \\ (N+1) \times N \end{array}$$

$$[a_k]_{k=0,1,2,3,4} = \begin{bmatrix} \frac{1}{2} a_{k,0} & & & & & \\ & a_{k,1} & & & & \\ & & a_{k,2} & & & \\ & & & \dots & & \\ & & & & a_{k,N-1} & \\ & & & & & \frac{1}{2} a_{k,N} \end{bmatrix} \quad \begin{array}{l} \text{(B5)} \\ (N+1) \times (N+1) \end{array}$$

APPENDIX B – Continued

$$\begin{bmatrix} \bar{a}_k \end{bmatrix}_{k=0,1,2} = \begin{bmatrix} 0 & & & & & \\ a_{k,1} & & & & & \\ & a_{k,2} & & & & \\ & & \dots & & & \\ & & & a_{k,N-1} & & \\ & & & & \frac{1}{2} a_{k,N} & 0 \end{bmatrix} \quad \begin{array}{l} \text{(B6)} \\ (N+1) \times (N+1) \end{array}$$

$$\begin{bmatrix} a_k^* \end{bmatrix}_{k=0,1,2} = \begin{bmatrix} a_{k,\frac{1}{2}} & & & & \\ & a_{k,\frac{3}{2}} & & & \\ & & \dots & & \\ & & & a_{k,N-\frac{1}{2}} & \end{bmatrix} \quad \begin{array}{l} \text{(B7)} \\ N \times N \end{array}$$

$$\begin{bmatrix} \hat{a}_0 \end{bmatrix} = \begin{bmatrix} a_{0,1} & & & & & \\ & a_{0,2} & & & & \\ & & \dots & & & \\ & & & a_{0,N-1} & & \\ & & & & \frac{1}{2} a_{0,N} & \\ & & & & & 0 \end{bmatrix} \quad \begin{array}{l} \text{(B8)} \\ (N+1) \times (N+1) \end{array}$$

in which

$$a_k(y) = \int_{x_L(y)}^{x_T(y)} D x^k dx \quad \text{(B9)}$$

$$D = \frac{E t h^2}{2(1 - \mu^2)} + \frac{2 E t^3}{12(1 - \mu^2)} \quad \text{(B10)}$$

APPENDIX B – Continued

and

E modulus of elasticity

t local thickness of either the upper or lower cover (airfoils are symmetrical)

h total local depth of wing

μ Poisson's ratio

The quantity $a_{k,0}$ is the value of a_k at the root, $y = 0$; $a_{k,\frac{1}{2}}$ is a_k at $y = \frac{1}{2} \cdot \epsilon$;

$a_{k,1}$ is a_k at $y = 1 \cdot \epsilon$; and $a_{k,N}$ is a_k at the tip. The wing tip is located at $y = N \cdot \epsilon$. (See fig. 3.) Several of the quantities which follow are defined in the same way.

The column vectors in equation (B1) describe the displacements and loading and are given by

$$\begin{bmatrix} \varphi_k \\ k=0,1,2 \end{bmatrix} = \begin{bmatrix} \varphi_{k,1} \\ \varphi_{k,2} \\ \cdot \\ \cdot \\ \cdot \\ \varphi_{k,N+1} \end{bmatrix} \quad (B11)$$

and

$$\begin{bmatrix} p_k \\ k=0,1,2 \end{bmatrix} = \begin{bmatrix} p_{k,1} \\ p_{k,2} \\ \cdot \\ \cdot \\ \cdot \\ p_{k,N} \\ 0 \end{bmatrix} \quad (B12)$$

in which

$$p_k(y) = \int_{x_L(y)}^{x_T(y)} p x^k dx \quad (B13)$$

This definition of p_k differs from that given in reference 11. As a result, there is a factor ϵ^4 in equation (B1) compared with a factor ϵ^3 in the equivalent equation (eq. (30)) of reference 14. The quantity p in equation (B13) is the intensity of the normal loading and has units of force per unit area.

In the present report the static normal loading used in the strength analysis of the wing is assumed to be a uniform loading; therefore,

$$p_k(y) = p \int_{x_L(y)}^{x_T(y)} x^k dx \quad (B14)$$

This completes the definitions of terms that appear in equation (B1). The deflections are calculated from equations (B1) and (2), and the strains are calculated from derivatives of the deflections (eqs. (4) to (6)). The stresses are calculated from the stress-strain relations given in equations (7) to (9).

Piston Theory for Unsteady Aerodynamics in Flutter Analysis

When the expression for $W(x,y,\tau)$ given in equation (1) is substituted into equation (10) the following equation results:

$$p(x,y,\tau) = \left[m\omega^2 w - 2\rho a \left(1 + \frac{\gamma + 1}{4} M \frac{\partial h}{\partial x} \right) \left(i\omega w + v \frac{\partial w}{\partial x} \right) \right] e^{i\omega t} \quad (B15)$$

where the positive direction is upward and

m mass of wing per unit area

ρ mass density of air

a speed of sound

γ ratio of specific heats

APPENDIX B - Continued

M Mach number, v/a

h total local depth of wing

v velocity

As noted in the body of the report (see eq. (11)), the unsteady loading $p(x,y,\tau)$ can be taken to the left side of the equilibrium equation (eq. (3) or (B1)), thereby allowing the equation to be written in the form

$$\left\{ \begin{bmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{bmatrix} + \epsilon^4 \begin{bmatrix} B_{00} & B_{01} & B_{02} \\ B_{10} & B_{11} & B_{12} \\ B_{20} & B_{21} & B_{22} \end{bmatrix} \right\} \begin{bmatrix} \varphi_0 \\ \varphi_1 \\ \varphi_2 \end{bmatrix} = 0 \quad (B16)$$

in which the components for A_{ij} are given in equation (B2) and the components for B_{ij} are as follows:

$$\left. \begin{aligned} [B_{00}] &= -\omega^2 [S_0] + 2\rho a i \omega \left([F_0] + \frac{v(\gamma+1)}{4a} [H_0] \right) \\ [B_{01}] &= -\omega^2 [S_1] + 2\rho a \left(i\omega [F_1] + v [F_0] \right) + \frac{2\rho v(\gamma+1)}{4} \left(i\omega [H_1] + v [H_0] \right) \\ [B_{02}] &= -\omega^2 [S_2] + 2\rho a \left(i\omega [F_2] + v [F_1] \right) + \frac{2\rho v(\gamma+1)}{4} \left(i\omega [H_2] + v [H_1] \right) \\ [B_{10}] &= -\omega^2 [S_1] + 2\rho a i \omega \left([F_1] + \frac{v(\gamma+1)}{4a} [H_1] \right) \\ [B_{11}] &= -\omega^2 [S_2] + 2\rho a \left(i\omega [F_2] + v [F_1] \right) + \frac{2\rho v(\gamma+1)}{4} \left(i\omega [H_2] + v [H_1] \right) \\ [B_{12}] &= -\omega^2 [S_3] + 2\rho a \left(i\omega [F_3] + 2v [F_2] \right) + \frac{2\rho v(\gamma+1)}{4} \left(i\omega [H_3] + 2v [H_2] \right) \\ [B_{20}] &= -\omega^2 [S_2] + 2\rho a i \omega \left([F_2] + \frac{v(\gamma+1)}{4a} [H_2] \right) \\ [B_{21}] &= -\omega^2 [S_3] + 2\rho a \left(i\omega [F_3] + v [F_2] \right) + \frac{2\rho v(\gamma+1)}{4} \left(i\omega [H_3] + v [H_2] \right) \\ [B_{22}] &= -\omega^2 [S_4] + 2\rho a \left(i\omega [F_4] + 2v [F_3] \right) + \frac{2\rho v(\gamma+1)}{4} \left(i\omega [H_4] + 2v [H_3] \right) \end{aligned} \right\} \quad (B17)$$

APPENDIX B – Continued

$$\begin{matrix} \begin{bmatrix} S_k \end{bmatrix} \\ k=0,1,2,3,4 \end{matrix} = \begin{bmatrix} S_{k,1} & & & & & \\ & S_{k,2} & & & & \\ & & \dots & & & \\ & & & S_{k,N-1} & & \\ & & & & \frac{1}{2} S_{k,N} & \\ & & & & & 0 \end{bmatrix} \quad \begin{matrix} \text{(B18)} \\ (N+1) \times (N+1) \end{matrix}$$

$$\begin{matrix} \begin{bmatrix} F_k \end{bmatrix} \\ k=0,1,2,3,4 \end{matrix} = \begin{bmatrix} F_{k,1} & & & & & \\ & F_{k,2} & & & & \\ & & \dots & & & \\ & & & F_{k,N-1} & & \\ & & & & \frac{1}{2} F_{k,N} & \\ & & & & & 0 \end{bmatrix} \quad \begin{matrix} \text{(B19)} \\ (N+1) \times (N+1) \end{matrix}$$

$$\begin{matrix} \begin{bmatrix} H_k \end{bmatrix} \\ k=0,1,2,3,4 \end{matrix} = \begin{bmatrix} H_{k,1} & & & & & \\ & H_{k,2} & & & & \\ & & \dots & & & \\ & & & H_{k,N-1} & & \\ & & & & \frac{1}{2} H_{k,N} & \\ & & & & & 0 \end{bmatrix} \quad \begin{matrix} \text{(B20)} \\ (N+1) \times (N+1) \end{matrix}$$

$$S_k = \int_{x_L(y)}^{x_T(y)} m x^k dx \quad \text{(B21)}$$

$$F_k = \int_{x_L(y)}^{x_T(y)} x^k dx \quad \text{(B22)}$$

$$H_k = \int_{x_L(y)}^{x_T(y)} \frac{\partial h}{\partial x} x^k dx \quad \text{(B23)}$$

APPENDIX B – Continued

This completes the definition of terms that appear in equation (B16). The determinant of the matrix of coefficients of φ_1 in equation (B16) is the flutter determinant (eq. (12)).

Mathematical Programming Techniques for Determining Minimum-Weight Design

In the interior penalty-function method denoted as "SUMT," the vector of design variables corresponding to the minimum value of the objective function is found by carrying out a sequence of minimizations of the P function. The P function $P(\vec{C}, r_k)$ is defined as

$$P(\vec{C}, r_k) = f(\vec{C}) + r_k \sum_j \frac{1}{g_j(\vec{C})} \quad (B24)$$

In equation (B24), $f(\vec{C})$ is the objective function, \vec{C} is the vector of design variables, and $r_k \sum_j \frac{1}{g_j(\vec{C})}$ is the penalty term in which $g_j(\vec{C}) > 0$ are the inequality constraints and r_k is the constraint weighting factor. The subscript k placed on r refers to the sequence of values that r assumes during the minimization process. Let \vec{C}_1 be the vector of design variables which satisfies the inequality constraints and minimizes the P function for an initial value of r . Let that value of r be denoted as r_1 . When \vec{C}_1 is found, the value of r is reduced to $r_2 < r_1$ and a search is carried out to find \vec{C}_2 , which is the vector of design variables that minimizes $P(\vec{C}, r_2)$. The starting point for that search is \vec{C}_1 . When \vec{C}_2 is found, the value of r is reduced to $r_3 < r_2 < r_1$. Starting with \vec{C}_2 , a search is carried out for \vec{C}_3 . This sequence of searches is continued until the penalty term contributes little to the P function which is then approximately equal to the objective function.

The manner in which the P function varies with r is illustrated in figure 10 for a simple one-dimensional design space. The single design variable is x , the objective function is $f(x)$, and the single inequality constraint is $g_1 \equiv x - b > 0$. The minimum value of P for $r = r_1$ is M_1 ; the minimum value of P for $r = r_2$ is M_2 , and so forth. Figure 10 is based upon figure 4(a) in reference 12.

For each value of r an unconstrained minimization is carried out. This procedure involves several two-step searches. First, it is necessary to generate a set of direction cosines \vec{Z} which defines a move direction in design-variable space. Second, a one-dimensional search is carried out to find the minimum value of P in the \vec{Z} direction.

APPENDIX B – Concluded

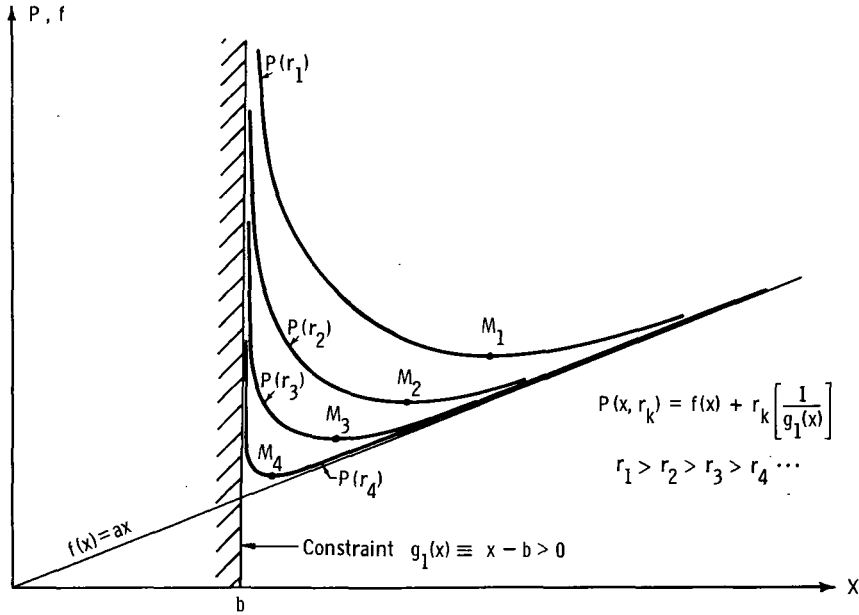


Figure 10.- Variation of P function with x for several values of the constraint weighting factor r . The minimum value of $P(r_k)$ is denoted by M_k . This is based upon figure 4(a) in reference 12.

In the present study, the direction-generating algorithm which was used almost exclusively was the Davidon-Fletcher-Powell method. Once a set of direction cosines \vec{Z} has been found, the design variables \vec{C} can be expressed as

$$\vec{C} = \vec{C}^* + S \cdot \vec{Z} \quad (\text{B25})$$

in which S is the distance traveled in the \vec{Z} direction and \vec{C}^* is the vector of design variables for $S = 0$. The search in the \vec{Z} direction is called a one-dimensional search because S is the only unknown.

In the one-dimensional search used to obtain the present results, the constraints $g_j(\vec{C})$ are approximated by polynomials in S . The polynomials never exceed second degree. Such a procedure is described in references 19 and 20. The flutter speed was found to be especially well-suited for polynomial approximation.

A more complete discussion of these optimization techniques is contained in references 12 and 17 to 23.

APPENDIX C

DESCRIPTION OF COMPUTER PROGRAM

The computer program SWIFT is written in FORTRAN IV for the CDC 6000 series computers at the Langley Research Center. The program also makes use of CDC library subroutine MATRIX (ref. 24). For these reasons the unmodified program may not be operational on other computer systems. A listing of the computer program is included so that mathematical operations can be examined in detail and so that modified programs can be developed. In the latter case this appendix can be considered to be an abbreviated user's document. It is assumed that the user is familiar with the material contained in the preceding sections of this report. A flow chart of the program is presented in figure 11.

SWIFT can be used to analyze or design a cantilever plate-like wing. The program calculates stress and deflections, natural frequencies, and flutter speeds and frequencies. When used in the design mode, the program can be used to find the minimum-weight wing cover-thickness distribution which satisfies flutter, strength, and minimum-gage design requirements. A six-design variable-strength design requires 4 to 6 minutes central-processing-unit execution (CPU) time. A six-design variable-flutter design requires 5 to 7 minutes CPU time. A strength-flutter (combined) design requires 6 to 8 minutes CPU time.

Description of Input

All of the input data are described in the main program. Most of the data are in namelist form. Additional description of input is given as follows:

The first card contains an optional word that is printed following the words "WING MATERIAL" in the output. "TITANIUM" is used in the sample problem in this appendix.

IOPT=1

This option is used to find a value of OMEGA, the flutter frequency, and V, the flutter speed, in the neighborhood of the flutter solution. Real and imaginary values of the complex determinant are calculated for values of OMEGA and V starting with OMINIT and VINIT and ending with OMFIN and VFIN in increments of DELOM and DELV. A solution occurs when both the real and imaginary parts of the flutter determinant are zero simultaneously. Natural frequencies can be calculated by setting RHOA, the air density, equal to zero. The effect of aerodynamic damping on the vibration frequencies can be explored by setting only V equal to zero.

APPENDIX C - Continued

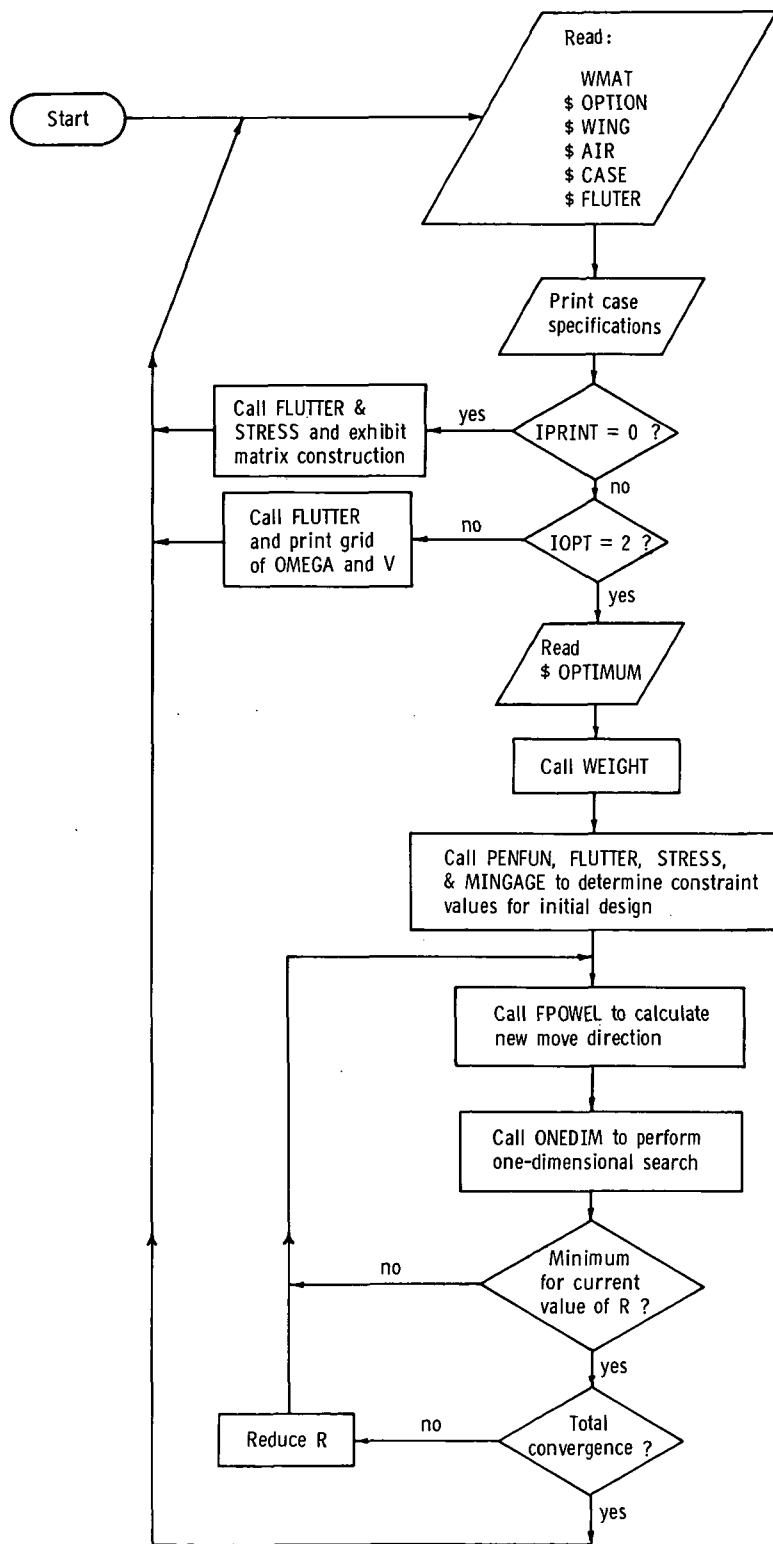


Figure 11.- Simplified flow chart of computer program SWIFT.

APPENDIX C – Continued

IOPT=2	Once the flutter-solution neighborhood has been established with IOPT=1, the program can be rerun with IOPT=2 to calculate flutter speed and frequency very accurately (within the limits of the theory) and to perform an optimization.
IPRINT=0	This option shows a printout of matrix construction. Matrix print statements require that NN, the number of spanwise stations, be set equal to 4. Final results can be used in deflection analysis.
IPRINT=1,2, or 3	These print options are used in the optimization mode.
ITHICK=2	The ITHICK=2 option is considered to give a more practical thickness distribution than the ITHICK=1 option. It is suggested that C(1) be set equal to the minimum-gage requirement. Limit active design variables to C(10), C(11), C(12), C(14), C(15), and C(16). Active design variables are those variables that are adjusted during the design process and are identified by nonzero DC(I). Set C(2) to C(9) and C(13) equal to zero.
DC(I)	These quantities are finite difference increments used to calculate derivatives with respect to the design variables C(I). If it is desired to hold a design variable fixed throughout the design process, set the DC(I) corresponding to that C(I) equal to zero.
NC	Subscript of the C(I) having the largest subscript. For example, if C(1), C(5), C(6), and C(14) are used to describe the thickness, then NC = 14.
IHP=0	Indicates that wing-depth-variation effects (sometimes called thickness effects) are not accounted for in piston theory.
IHP=1	Indicates that wing-depth-variation effects are accounted for in piston theory.
NN	Number of semispan stations, not counting the root station, where 10 is maximum and 6 to 8 appear to be adequate.
NCS	Number of chordwise stations used in numerical integration divided by 10. For example, when NCS=1, 10 stations are used by subroutine MGAUSS.
DELOM, DELV	Increments for OMEGA and V. Although these increments can be large for IOPT=1, they must be small for IOPT=2 where they are used as finite difference increments in a Newton-Raphson search to converge on OMEGA and V. For example, DELOM=1 and DELV=1000 are reasonable numbers for the IOPT=1 option.

APPENDIX C – Continued

However, for the IOPT=2 option these increments must be reduced to the order of DELOM=0.0001 and DELV=0.1.

- R** Constraint weighting factor used in penalty-function formulation. Choose initial R so that the initial value of the penalty function is of the same order of magnitude as the objective function, which is the wing weight. In the program, both the stress and minimum-gage constraints have a built-in additional multiplier of 30. This additional multiplier was inserted because both the stress and minimum-gage constraints are averaged over the surface.
- RDC** Factor by which R is divided at the end of each unconstrained search cycle. Numbers on the order of 5 to 15 are reasonable. The larger the value of RDC, the fewer the number of unconstrained search cycles. However, large values of RDC may also lead to false indications of convergence. Adjust RDC so that at least three unconstrained searches are carried out.
- DESCON** Three logical variables. The values of each are T or F indicating true or false. The three variables refer to the three constraints being considered: flutter, stress, and minimum gage. For example, TFT indicates that the user wishes to consider the flutter and minimum-gage constraints but wishes to ignore the stress constraint. When a constraint is ignored, no analysis is performed to check that constraint.

Operational Hints

Flutter.- There are many values of OMEGA and V which satisfy the equations obtained by setting the flutter determinant equal to zero. During a computer run in the optimization mode IOPT=2, the program locks in on a flutter solution and follows it throughout the optimization process. To obtain the initial solution – that is, OMIN and VIN – the user runs the IOPT=1 option and chooses the solution which has the lowest flutter speed. For the final design, however, the program may not be examining the flutter solution corresponding to the lowest flutter speed.

The only way to guarantee that the final design does satisfy the flutter constraint is to rerun the IOPT=1 option using the final values of C(I). Although experience indicates that this procedure is seldom required, it is recommended until a user becomes familiar with the particular class of problems on which he is working.

Convergence.- Convergence to a global extremum is of prime concern to the user. In general, the best way to increase one's confidence in a particular solution is to rerun the problem with a different initial design, different initial value of R, and/or a different

APPENDIX C – Continued

value of RDC. A technique is presented in reference 25 for warping design variable space in such a way that the second run will have a greater chance of locating another minimum, if one exists.

The smaller the final values of a constraint function $G(I)$, the closer the design is to that constraint. The following values of the constraint functions $G(I)$ are reasonable for convergence: $G(1) = 0.03$, $G(2) = 0.15$, and $G(3) = 0.15$. Designs having a value of $G(I)$ larger than those indicated should be examined for false convergence.

Scaling of flutter determinant. - The object of subroutine DEUPPS is to evaluate the complex flutter determinant as rapidly as possible. Because of the overriding need for speed, it was decided to eliminate certain scaling logic that would normally prevent overflow and underflow. Scaling must be carried out by the user. The elements in the complex flutter matrix are scaled in statements 101, 102, 103, and 104 in subroutine FLUTTER.

Program listing and sample output. - The remaining part of this appendix contains a listing of the program denoted as "SWIFT," a printout of a computer run used to determine the flutter-solution neighborhood, and a printout of an optimization run.

APPENDIX C - Continued

```

PROGRAM BIGBOY(INPUT=201,OUTPUT=201,TAPE5=INPUT)

C THIS IS THE MAIN PROGRAM.
C ITS FUNCTION IS TO READ AND PRINT MOST OF THE INPUT VARIABLES, SET
C VALUES FOR A NUMBER OF CONSTANTS, AND INITIATE THE CALLING OF
C OTHER SUBROUTINES IN AN APPROPRIATE SEQUENCE AS SPECIFIED BY
C THE INPUT OPTIONS.

1 FORMAT(1H1 /5X*DETERMINATION OF THICKNESS DISTRIBUTION FOR MINI
IMUM WEIGHT WING*/
25X*SATISFYING PRESCRIBED FLUTTER AND STRENGTH CRITERIA.*/
35X*REFERENCE - NACA TN-3640, STEIN-SANDERS, JUNE,1956.*/
45X*DEXTER-STROUD RDK288 MAY,1969.*///
55X*DATE#A10)
100 FORMAT(A10)
502 FORMAT(/103X*XXX*/103X*XXX*/ * WING DESCRIPTION*,86X*XXX*,
110X*FIXED BOUNDARY*/102X*.XXX*,10X*CONDITION*/
15X*ASPECT RATIO.....AR =E16.8,52X
1*. XXX*/
25X*SURFACE AREA.....AREA =E16.8,7H IN.**2,44X
2*. XXX*/
35X*L.E. SWEEP ANGLE.....ANG =E16.8* DEGREES*,42X
3*. XXX*/
45X*SEMISPAN.....SS =E16.8* IN.,45X
4*. XXX*/
55X*ROOT CHORD.....RC =E16.8* IN.,44X
5*. XXX*)
512 FORMAT(5X*DEPTH RATIO.....DR =E16.8,47X
6*. XXX*/
75X*WING MATERIAL.....*A1C,54X
7*. XX ROOT CHORD =#F6.1* IN.*/
85X*MASS DENSITY.....RHOW =E16.8,23H (LB.*SEC.**2)/(IN.**4)
822X
8*. XXX*/
95X*YOUNGS MODULUS.....E =E16.8* PSI*,40X
9*. XXX*/
15X*POISSONS RATIO.....MU =E16.8,43X
1*. XXX*/
291X*. XXX*/90X*.#F3.0* DEG. XXX*)
503 FORMAT(// * EQUATION FOR THICKNESS - TBAR (INCHES)*/)
5031 FORMAT(* TBAR =#3X41HC( 1) + C( 2)*A + C( 3)*A**2 + C( 4)*A**3/
18X46H+(C( 5) + C( 6)*A + C( 7)*A**2 + C( 8)*A**3)*B/

```

```

18X49H+(C( 9) + C(10)*A + C(11)*A**2 + C(12)*A**3)*B**2/
18X49H+(C(13) + C(14)*A + C(15)*A**2 + C(16)*A**3)*B**3//
112X43HWHERE A = 1-Y/SS AND B = 1-2*X/(LE+TE)//
126X12H(ITHICK = 1)
5032 FORMAT(* TEAR =3X41HC( 1) + C( 2)*A + C( 3)*A**2 + C( 4)*A**3/
18X46H+(C( 5) + C( 6)*A + C( 7)*A**2 + C( 8)*A**3)*B/
18X52H+(C( 9) + C(10)*A + C(11)*A**2 + C(12)*A**3)*(1-8*B)/
18X54H+(C(13) + C(14)*A + C(15)*A**2 + C(16)*A**3)*(1-8*B)*B//
112X43HWHERE A = 1-Y/SS AND B = 1-2*X/(LE+TE)//
126X12H(ITHICK = 2)
531 FORMAT(/5X*DESIGN VARIABLES*,17X*FINITE DIFFERENCE INCREMENTS*,
16X*ANALYTICAL DERIVATIVES*
2 11X1H*,* INDICATES ACTIVE*/107X*DESIGN VARIABLES*)
533 FORMAT(5X*C(*I2*) =*E16.8,10X*CC(*I2*) =*E16.8,10X*AD(*I2*) =*E16.
18)
534 FORMAT(2X1H*,2X*C(*I2*) =*E16.8,10X*DC(*I2*) =*E16.8,10X*AD(*I2*)
1=*E16.8)
504 FORMAT(* AIR PROPERTIES*75X
1*. . . . .XXX*/103X*XXX*/
15X*SPEED OF SOUND.....A =*E16.8* IN./SEC. =*E16.8* FT./SEC.*
2 5X*SEMI SPAN = XXX*,21H LEADING EDGE = A+B*Y/
25X*MASS DENSITY.....RHOA =*E16.8,23H (LB.*SEC.**2)/(IN.**4),
218XF5.1,* IN. XXX A =*E16.8/
35X*CCRRESPONDING ALTITUDE... =*E16.8* FT.*60X
3*B =*E16.8)
5 FORMAT(/** STATIONS**/
15X*NUMBER OF SEMISPAN STATIONS.....NN =*I3/
25X*NUMBER OF CHORDWISE STATIONS = (10 X NCS)....NCS =*I3)
22 FORMAT(/** RANGE AND INCREMENT OF ITERATION**/
15X*OMEGA INITIAL.....OMINIT =*E16.8* RADIANS/SEC.*
25X*OMEGA INCREMENT.....DELOM =*E16.8* RADIANS/SEC.*
35X*OMEGA FINAL.....CMFIN =*E16.8* RADIANS/SEC.*
45X*V INITIAL.....VINIT =*E16.8* IN./SEC.*
55X*V INCREMENT.....CELV =*E16.8* IN./SEC.*
55X*V FINAL.....VFIN =*E16.8* IN./SEC.*
50 FORMAT(/** INPUT OPTIONS**/
15X* IOPT =*I2,5X*1 - ITERATE ON OMEGA AND V, 2 - COMPLEX SEARCH*
1/
15X* IPRINT =*I2,5X*0 - MATRIX CONSTRUCTION, 1 - SHORT PRINTOUT, 2
1- INTERMEDIATE PRINTOUT, 3 - EXTENDED PRINTOUT*/
75X*ISNDWCH =*I2,5X*0 - SCLID WING, 1 - SANDWICH CONSTRUCTION*/
85X* IHP =*I2,5X*0 - THICKNESS VARIATION EFFECTS NOT ACCOUNTED F
80R IN PISTON THEORY, 1 - THICKNESS EFFECTS ARE USED*/

```

APPENDIX C - Continued

8)	REAL MU	8600000
	DIMENSION RESULT(2)	8700000
	DIMENSION H(16)	8800000
	COMPLEX AAC	8900000
	COMMON	9000000
	1/BLK1/NN,NN1,NN2,NN22,NN23,NN33,MCS,NCS,NMAX	9100000
	1/BLK20/IFIRST,IFAIL,NOCONV,NOS	9200000
	3/BLK3/ANG,RHOW,E,MU,C(16),SS,EPS	9300000
	4/BLK4/A,RHOA,IHP	9400000
	5/BLK5/OMEGA1,DELOM,OMFIN,V1,DELV,VFIN	9500000
	1/BLK6/IOPT,IPRINT,ISNDWCH	9600000
	1/BLK12/VCR,R,NC,H,DESCON(3)	9700000
	3/BLK13/Y,C1,C2	9800000
	7/BLK17/NEWR	9900000
	1/BLK50/RHOF	10000000
	8/BLK18/DR	10100000
	1/BLK28/CSAV(16),DC(16),D(16),DIRCOS(16),DM1,X	10200000
	2/BLK32/IG	10300000
	3/BLK36/C1A,C1B,C2A	10400000
	3/BLK40/AD(16)	10500000
	1/NEWBLK/AAC(33,33),AA(33,33),S(5,11),F(5,11),HP(5,11)	10600000
	1/BLKNPS/AREA	10700000
	1/NPSBLK/ITHICK	10800000
	NAMELIST/OPTION/IOPT,IPRINT,ISNDWCH,ITHICK	10900000
	NAMELIST/WING/ANG,RHOW,E,MU,WTF,AR,AREA,DR,NC,C,DC	11000000
	NAMELIST/AIR/A,RHOA,CORALT,IHP	11100000
	NAMELIST/CASE/NN,NCS	11200000
	NAMELIST/FLUTER/OMINIT,DELCM,OMFIN,VINIT,DELV,VFIN	11300000
		11400000
		11500000
		11600000
		11700000
		11800000
		11900000
		12000000
		12100000
		12200000
		12300000
		12400000
		12500000
		12600000
		12700000
		12800000

C	INPUT DATA CARDS
C	1 - WING MATERIAL (FORMAT A10)
C	2 - \$OPTCN
C	IOPT 1-ITERATE OMEGA AND V, 2-COMPLEX SEARCH,OPTIMIZATION
C	IPRINT 0-MATRIX CCNSTRUCTION - FOR TROUBLE-SHOOTING,
C	USE NN=4 ONLY
C	1-SHORT PRINTOUT - INITIAL CASE PLUS FINAL RESULT

APPENDIX C - Continued

C	2-INTERMEDIATE PRINTOUT - FINAL RESULTS OF FLETCHER-POWELL AND ONE-DIMENSIONAL SEARCH	12900000
C	3-EXTENDED PRINTOUT - FOR TROUBLE-SHOOTING.	13000000
C	GIVES FINITE DIFFERENCE AND ONE-DIMENSIONAL SEARCH DETAILS	13100000
C	0-SOLID WING, 1-SANDWICH CONSTRUCTION	13200000
C	1-GENERAL 16-TERM THICKNESS EQUATION	13300000
C	2-MODIFIED THICKNESS EQUATION - THICKNESS IS EQUAL TO C(1) ALONG LEADING EDGE, TRAILING EDGE, AND TIP	13400000
C	LEADING EDGE SWEEP ANGLE - DEGREES	13500000
C	MASS DENSITY - LB.*SEC.**2/IN.**4	13600000
C	YOUNGS MODULUS - PSI	13700000
C	POISSONS RATIO	13800000
C	WEIGHT OF FUEL IN ONE WING - POUNDS	13900000
C	ASPECT RATIO - SPAN**2/TOTAL PLANFORM SURFACE AREA	14000000
C	SURFACE AREA OF SEMISPAN - IN.**2	14100000
C	DEPTH RATIO - RATIO OF MAXIMUM DEPTH OF AIRFOIL TO LOCAL CHORD	14200000
C	NUMBER OF DESIGN VARIABLES (16 MAXIMUM)	14300000
C	DESIGN VARIABLES (16 MAXIMUM)	14400000
C	FINITE DIFFERENCE INCREMENTS (16 MAXIMUM)	14500000
C	4 - \$AIR	14600000
C	A SPEED OF SOUND - INCHES/SECOND	14700000
C	RHOA MASS DENSITY - LB.*SEC.**2/IN.**4	14800000
C	CORALT CORRESPONDING ALTITUDE - FEET	14900000
C	IHP O-NO HPRIME TERMS IN COMPLEX MATRIX, 1-HPRIME TERMS	15000000
C	5 - \$CASE	15100000
C	NN NUMBER OF SEMISPAN STATIONS (10 MAXIMUM)	15200000
C	NCS NUMBER OF CHORDWISE STATIONS/10	15300000
C	6 - \$FLUTER	15400000
C	OMINIT OMEGA INITIAL - RADIAN/SECOND	15500000
C	CELOM OMEGA INCREMENT - RADIAN/SECOND	15600000
C	CMFIN OMEGA FINAL (IF IOPT=1)	15700000
C	VINIT V INITIAL - INCHES/SECOND	15800000
C	DELV V INCREMENT - INCHES/SECOND	15900000
C	VFIN V FINAL (IF IOPT=1)	16000000
C		16100000
C		16200000
C		16300000
C		16400000
C		16500000
C		16600000
C		16700000
C		16800000
C		16900000
C		17000000
C		17100000

38

APPENDIX C - Continued

```

NN2 = NN+2
NN22 = 2*NN+2
NN23 = 2*NN+3
NN33=3*NN+3
CALL ANDER
PRINT 1, RESULT(1)
PRINT 502, AR, AREA, ANG, SS, RC
PRINT 512, DR, WMAT, RC, RHOH, E, MU, ANG
PRINT 504, A, AFT, RHOA, SS, CIA, CORALT, C1B
IF (ISNDWCH.EQ.0) GO TO 400
Y = 0.
CALL CEE
A1 = 2./3.*DR*(C2-C1)**2
Y = SS
CALL CEE
A2 = 2./3.*DR*(C2-C1)**2
VOL = 1./3.*(A1+A2+SQR(A1*A2))*SS
RHOF = WTF/(VOL*386.)
PRINT 505, WTF, VOL, RHOF
505 FORMAT(// * FUEL PROPERTIES//
15X*WEIGHT OF FUEL.....WTF =*E15.8* LB.*/
25X*VOLUME OF WING.....VOL =*E15.8*H IN.**3/
35X*DENSITY OF FUEL.....RHOF =*E16.8,23H (LB.*SEC.**2)/(IN.**4)/
1)
400 CONTINUE
PRINT 503
IF (ITHICK.EQ.1) PRINT 5031
IF (ITHICK.EQ.2) PRINT 5032
PRINT 531
DO 532 I=1,NC
IF (OC(I).EQ.0.) GO TO 535
PRINT 534, I,C(I),I,DC(I),I,AD(I)
GO TO 532
535 CONTINUE
PRINT 533, I,C(I),I,DC(I),I,AD(I)
532 CONTINUE
600 CONTINUE
PRINT 5, NN,NCS
PRINT 50, IOPT,IPRINT,ISNDWCH,IHP
CALL FMAT
CALL HMAT
CALL WEIGHT(WT)
PRINT 601, WT

```

```

601 FORMAT(////////* INITIAL WEIGHT =*E16.8,* POUNDS*//)
      IF(IPRINT.EQ.0) GO TO 201
      IF(IOPT.EQ.2) GO TO 202
      NOCONV = 0
      NEWR = 1
      PRINT 22, OMINIT,DELCM,OMFIN,VINIT,DELV,VFIN
      CALL FLUTTER
      GO TO 1000
201 CONTINUE
      NOCONV = 0
      NEWR = 0
      CALL FLUTTER
      CALL STRESS(G2)
      GO TO 1000
202 CONTINUE
      CALL METHOD
      GO TO 1000
      END
25800000
25900000
26000000
26100000
26200000
26300000
26400000
26500000
26600000
26700000
26800000
26900000
27000000
27100000
27200000
27300000
27400000
27500000

```

APPENDIX C - Continued

```

SUBROUTINE MGAUSS(A,B,N,SUM,FUNC,FOFX,NUMBER)
***** DOCUMENT DATE 08-01-68 SUBROUTINE REVISED 08-01-68 *****D1.1 2
C      THIS SUBROUTINE INTEGRATES FROM ZERO TO ONE
      DIMENSION U(5),R(5),SUM(1),FOFX(1)
      COMMON
      2/BLK20/IFIRST,IFAIL,NOCONV
      DO 1 LL=1,NUMBER
        1 SUM(LL)=0.0
          IF(A.EQ.8)RETURN
          U(1)=.425562830509184
          U(2)=.283302302935376
          U(3)=.160295215850488
          U(4)=.067468316655508
          U(5)=.013046735741414
          R(1)=.147762112357376
          R(2)=.134633359654998
          R(3)=.109543181257991
          R(4)=.074725674575290
          R(5)=.033335672154344
          FINE= N
          DELTA=FINE/(8-A)
          DO 3 K=1,N
            XI=K-1
            FINE=A+XI/DELTA
            DO 2 II= 1,5
              UU=U(II)/DELTA+FINE
              CALL FUNC (UU,FOFX)
              IF(IFAIL.EQ.1) RETURN
            DO 2 JOYBOY=1,NUMBER
              2 SUM(JOYBOY)=R(II)*FOFX(JOYBOY)+ SUM(JOYBOY)
            DO 3 JJ=1,5
              UU=(1.0-U(JJ))/DELTA+FINE
              CALL FUNC (UU,FOFX)
              IF(IFAIL.EQ.1) RETURN
            DO 3 NN=1,NUMBER
              3 SUM(NN)=R(JJ)*FOFX(NN)+SUM(NN)
            DO 7 IJK= 1,NUMBER
              7 SUM(IJK)=SUM(IJK)/DELTA
            RETURN
          END

```

```

SUBROUTINE ANDER
C ANALYTICAL DERIVATIVES OF WEIGHT WITH RESPECT TO DESIGN VARIABLES
C C(I)
REAL MU
COMMON
3/BLK3/ANG,RHOW,E,MU,C(16),SS,EPS
3/BLK36/C1A,C1B,C2A
3/BLK40/AD(16)
1/NPSBLK/ITHICK
SIXINV = 1./6.
THREINV = 2.*SIXINV
RHO2 = RHO*772.
CCS = (C2A-C1A)*SS
CBS = C1B*SS**2
DO 100 I=1,16
100 AD(I) = 0.
AD(1) = RHO2*(CCS-.5*CBS)
AD(2) = RHO2*(.5*CCS-SIXINV*CBS)
AD(3) = RHO2*(THREINV*CCS-.5*SIXINV*CBS)
AD(4) = RHO2*(.25*CCS-.05*CBS)
C FOLLOWING 4 ANALYTIC DERIVATIVES CORRECT ONLY FOR C2=TE=0
AD(9) = THREINV*AD(1)
AD(10) = THREINV*AD(2)
AD(11) = THREINV*AD(3)
AD(12) = THREINV*AD(4)
IF(ITHICK.EQ.1) RETURN
AD(10) = AD(2) - AD(10)
AD(11) = AD(3) - AD(11)
AD(12) = AD(4) - AD(12)
RETURN
END

```

APPENDIX C - Continued

```

SUBROUTINE METHOD
10 FORMAT(///** WT = WEIGHT IN POUNDS =*E16.8/* PF = PENALTY FUN
1CTION =*E16.8/* PFUNC = WT + PF =*E16.8/** CONSTANTS I -*
113/(4E20.8))
15 FORMAT(/125(1H*))
18 FORMAT(///** INPUT DATA FOR SEARCH ROUTINE**
15X*NUMBER OF DESIGN VARIABLES...NC =*I3/
15X*
15X*R REDUCTION FACTOR.....RDC =*E16.8/
17 FORMAT(///** INPUT DATA FOR STRESS**
35X*UNIFORM PRESSURE LOADING...PLCAD =*E16.8* PSI*/
45X*YIELD STRESS.....YCR =*E16.8* PSI*)
19 FORMAT(///** INPUT FOR MINGAGE**
15X*MINIMUM GAGE.....TBARMIN=*E16.8* INCHES*)
20 FORMAT(3L5)
21 FORMAT(///** DESIGN CONSTRAINTS CONSIDERED*/
15X*FLUTTER -*L2/5X* STRESS -*L2/5X*MINGAGE -*L2)
28 FORMAT(///** INPUT DATA FOR FLUTTER**
15X*OMEGA INITIAL.....OMINIT =*E16.8* RADIANS/SEC.*//
25X*OMEGA INCREMENT.....DELOM =*E16.8* RADIANS/SEC.*//
35X*V INITIAL.....VINIT =*E16.8* IN./SEC. =*E16.8* FT.
3/SEC.*//
45X*V INCREMENT.....DELV =*E16.8* IN./SEC. =*E16.8* FT.
4/SEC.*//
55X*V CRITICAL.....VCR =*E16.8* IN./SEC. =*E16.8* FT.
5/SEC.*//
27 FORMAT(///** FLETCHER - POWELL SEARCH ROUTINE*/
1* PERTURBED CASES*)

DIMENSION H(16)
LOGICAL DESCON
REAL MU
COMMON
2/BLK2/CMEGA,V
3/BLK3/ANG,RHOW,E,MU,C(16),SS,EPS
5/BLK5/CMEGA1,DELOM,OMFIN,V1,DELV,VFIN
1/BLK6/ICPT,IPRINT,ISNDWCH
8/BLK8/KDERIV
1/BLK12/VCR,R,NC,H,DESCON(3)
2/BLK20/IFIRST,IFAIL,NOCONV,NC5
2/BLK14/YCR,IY

```

APPENDIX C - Continued

```

1/BLK15/PLOAD
7/BLK17/NEW
3/BLK25/TBARMIN
1/BLK28/CSAV(16),DC(16),D(16),DIRCOS(16),DM1,PF
3/BLK33/G1,G2,G3
5/BLK35/OMEGAO,V0
7/BLK37/G11,G22,G33
3/BLK40/AD(16)
4/BLK41/WT
4/BLK44/RDC
  NAMELIST/OPTIMUM/RDC,R,VCR
  NAMELIST/STRSS/PLOAD,YCR
  NAMELIST/GAGE/TBARMIN
  READ OPTIMUM
  READ 20, (DESCON(J),J=1,3)
  IF(DESCON(2)) READ STRSS
  IF(DESCON(3)) READ GAGE
  PRINT 18, NC,R,RDC
  PRINT 21, (DESCON(J),J=1,3)
  VIF = V1/12,
  DELVF = DELV/12,
  VCRF = VCR/12,
  IF(DESCON(1)) PRINT 28, CMGAL,DELOM,V1,VIF,DELV,VCR,VCRF
  IF(DESCON(2)) PRINT 17, PLOAD,YCR
  IF(DESCON(3)) PRINT 19, TBARMIN
  DO 302 J=1,NC

C  EVALUATE NCMINAL WEIGHT
302 CSAV(J) = C(J)
   IFAIL = 0
   NFWR = 1
   CALL WEIGHT(WT)

C  EVALUATE PF (PENALTY FUNCTION)
  KDERIV = 0
  IFIRST = 1
  CALL PENFUN(PF)
  IF(IFAIL.EQ.1) GO TO 350
  OMEGAO = OMEGA
  V0 = V
  G11 = G1
39100000
39200000
39300000
39400000
39500000
39600000
39700000
39800000
39900000
40000000
40100000
40200000
40300000
40400000
40500000
40600000
40700000
40800000
40900000
41000000
41100000
41200000
41300000
41400000
41500000
41600000
41800000
41900000
42000000
41700000
42100000
42200000
42300000
42400000
42500000
42600000
42700000
42800000
42900000
43000000
43100000
43200000
43300000

```

APPENDIX C - Continued

```

G22 = G2
G33 = G3
OMEGA1 = OMEGA
V1 = V
PFUNC=WT+PF
PRINT 10, WT, PF, PFUNC, NC, (C(J), J=1, NC)
DO 100 J=1, NC
100 CSAV(J) = C(J)
IFIRST = 2

IF(IPRINT.EQ.3) PRINT 27
CALL FPCWEL
RETURN

550 PRINT 50
50 FORMAT(/* FAILURE WITH INITIAL DESIGN*)
STOP
END
43400000
43500000
43600000
43700000
43800000
43900000
44000000
44100000
44200000
44300000
44400000
44500000
44600000
44700000
44800000
44900000
45000000

```

APPENDIX C - Continued

```

SURROUTINE FPOWER
C  DAVIDON-FLETCHER-POWELL DIRECTION GENERATOR PLUS CONVERGENCE TESTS
1  FORMAT(/5X*PF =*E16.8)
3  FORMAT(/* PF PLUS =*E16.8/(5E25.8))
4  FORMAT(/* PF MINUS =*E16.8/(5E25.8))
15 FORMAT(/125(1H*))
111 FORMAT(125(1H*))
110 FORMAT(/* MOVE DIRECTION DATA FOR ONE-DIMENSIONAL SEARCH*/)
14 FORMAT(/5X*FIRST DERIVATIVES/(4X8E16.8))
17 FORMAT(/5X*DIRECTION COSINES/(4X8E16.8))
19 FORMAT(/5X*CSAV/(4X8E16.8))
18 FORMAT(/5X*PFUNC = WT+PF =*E16.8,10X*DERIVATIVE OF PFUNC IN MOVE D
1 IRECTION =*E16.8)
109 FORMAT(//)
112 FORMAT(/* ONE DIMENSIONAL SEARCH - FLETCHER-POWELL METHOD*/)
33 FORMAT(/* DERIVATIVE OF PFUNC IN MOVE DIRECTION IS POSITIVE - REIN
1 IIALIZE HH MATRIX*)
36 FORMAT(/* DERIVATIVE OF PFUNC IN MOVE DIRECTION IS POSITIVE WITH I
1 INITIAL HH MATRIX - PROCEED TO CTEST*)
62 FORMAT(/* GRADIENT CONVERGENCE TEST SATISFIED*)
76 FORMAT(/* Q NEGATIVE - REINITIALIZE HH MATRIX*)
81 FORMAT(/* QUADRATIC CONVERGENCE TEST SATISFIED*)
101 FORMAT(/* MINIMUM MOVE SIZE CONVERGENCE TEST SATISFIED*)
301 FORMAT(/* ESTIMATED MINIMUM WEIGHT LESS THAN 2 PERCENT LOWER THAN
1 PRESENT WEIGHT*/* DESIGN CONSIDERED CONVERGED*)
311 FORMAT(/* CURRENT VALUE OF R IS*E16.8)
505 FORMAT(/* PF =*E16.8,* WT =*E16.8,* S =*E16.8/* CONSTANTS*/
1(4E16.8))
921 FORMAT(/* CTEST =*E16.8)

REAL MU,M1,I,IPLUS,IMINUS
REAL M2,MPLUS
LOGICAL DESCON
DIMENSION H(16)
DIMENSION SIG(16),Y(16),SIGT(16),YT(16),A(16,16),B(16,16),
1HH(16,16),D2(16)
COMMON
3/RLK3/ANG,RHOW,E,MU,C(16),SS,EPS
1/RLK6/IOPT,IPRINT,ISNDWCH
8/RLK9/KDERIV

```


APPENDIX C - Continued

```

1/BLK12/VCR,R,NC,H,DESCON(3)
7/BLK17/NEW
2/BLK20/IFIRST,IFAIL,NOCONV,NOS
1/BLK28/CSAV(16),DC(16),D(16),DIRCOS(16),DM1,I
1/BLK31/DCR,OMPLUS,VPLUS
3/BLK33/G1,G2,G3
4/BLK34/GP1,GP2,GP3
3/BLK40/AD(16)
4/BLK41/WT
5/BLK42/M1
1/BLK43/S
4/BLK44/RDC
EQUIVALENCE (I,PF),(SIGT,YT),(A,B)

MULT = 20
ISCLR = 24
ITRANS = 0
ISUB = 22
IADD = 21

C      CENTRAL DIFFERENCE CALCULATION OF FIRST DERIVATIVES

10 CONTINUE
KOUNTR = 0
KDERIV = 1
DO 11 K=1,NC
C      SET D = 0 FOR INACTIVE DESIGN VARIABLES
      IF(DC(K).EQ.0.) GO TO 12
      C(K) = CSAV(K)+DC(K)
      CALL PENFUN(IPLUS)
      IF(IFAIL.EQ.1) GO TO 501
      IF(IPRINT.NE.3) GO TO 200
      PRINT 3, IPLUS,(C(J),J=1,NC)
      PRINT 15
200 CONTINUE
      C(K) = CSAV(K)-DC(K)
      CALL PENFUN(IMINUS)
      IF(IFAIL.EQ.1) GO TO 501
      IF(IPRINT.NE.3) GO TO 201
      PRINT 4, IMINUS,(C(J),J=1,NC)
      PRINT 15
201 CONTINUE
      D(K) = (IPLUS-IMINUS)/(2.*DC(K))+AD(K)

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C(K) = CSAV(K)
GO TO 11
12 D(K) = C.
11 CONTINUE
GO TO 20
501 DO 511 J=1,NC
511 DC(J) = DC(J)/2.
IFAIL = 0
GO TO 10

C HH MATRIX INITIALIZED TO IDENTITY MATRIX

20 CONTINUE
NCA = NC
DO 48 J=1,NC
IF(DC(J).EQ.0.) NCA = NCA-1
48 CONTINUE
DO 21 K=1,NCA
DO 21 J=1,NCA
HH(J,K) = 0.
IF(J.EQ.K) HH(J,K) = 1.
21 CONTINUE
KOUNT = 0
49 CONTINUE

C REDUCE D VECTOR TO NCA (ACTIVE DESIGN VARIABLES ONLY)

NCA = NC
IF(NC.EQ.1) GO TO 30
J = NC-1
46 CONTINUE
IF(DC(J).NE.0.) GO TO 37
NCA = NCA-1
DO 38 JJ=J,NCA
38 D(JJ) = D(JJ+1)
37 CONTINUE
J = J-1
IF(J.EQ.0) GO TO 47
GO TO 46
47 CONTINUE
IF(DC(NC).EQ.0.) NCA = NCA-1
30 KOUNT = KOUNT+1
KOUNTR = KOUNTR+1

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APPENDIX C - Continued

```

SCLR = -1.
CALL MATRIX(ISCLR,NCA,NCA,G,SCLR,O,HH,16,HH,16)
CALL MATRIX(MULT,NCA,NCA,1,HH,16,D,16,H,16)
CALL MATRIX(ISCLR,NCA,NCA,O,SCLR,O,HH,16,HH,16)

C  EXPAND D AND H VECTORS TO NC
      IF(NC.EC.1) GO TO 53
      K = NC-1
      DO 39 J=1,K
      IF(DC(J).NE.O.) GO TO 39
      J1 = NCA
      DO 45 JJ=J,NCA
      D(J1+1) = D(J1)
      H(J1+1) = H(J1)
      J1 = J1-1
45  CONTINUE
      NCA = NCA+1
      D(J) = O.
      H(J) = O.
39  CONTINUE
      IF(DC(NC).EQ.O.) D(NC) = H(NC) = O.
53  CONTINUE
      SUM = O.
      DO 31 K=1,NC
31  SUM = SUM+H(K)**2
      SUM = SORT(SUM)
      DCR = O.
      DO 32 K=1,NC
      DIRCOS(K) = H(K)/SUM
      IF(ABS(DIRCOS(K)).LT.DCR) GO TO 32
      KK = K
      DCR = ABS(DIRCOS(K))
32  CONTINUE

C  DCR EQUALS THE FINITE DIFFERENCE INCREMENT FOR THE DIRECTION HAVING
C    THE LARGEST DIRECTION COSINE (DIRCOS(K))
      DCR = DC(KK)
35  CONTINUE

C  SHORT STEP IN MOVE DIRECTION

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APPENDIX C - Continued

```

DO 34 K=1,NC
34 C(K) = CSAV(K)+DIRCOS(K)*DCR
   KDERIV = 1
   IF(IPRINT.EQ.3) PRINT 2
2  FORMAT(* SHORT STEP IN MOVE DIRECTION*)
   CALL PENFUN(IPLUS)
   IF(IFAIL.EQ.1) GO TO 503
   GP1 = G1
   GP2 = G2
   GP3 = G3
   IF(IPRINT.NE.3) GO TO 202
   PRINT 3, IPLUS,(C(J),J=1,NC)
   PRINT 17, (DIRCOS(J),J=1,NC)
202 CONTINUE
   DM1 = (IPLUS-1)/DCR
   DO 321 K=1,NC
321 DM1 = DM1+DIRCOS(K)*AD(K)
   IF(DM1.LT.0.) GO TO 41
   IF(IPRINT.EQ.3) PRINT 15
   IF(KOUNT.EQ.1) GO TO 322
   IF(IPRINT.GT.1) PRINT 33
   GO TO 20
322 IF(IPRINT.GT.1) PRINT 36
   GO TO 300
503 CONTINUE
   DCR = DCR/2.
   IFAIL = 0
   GO TO 35
41 CONTINUE
   M1 = PF+WT
   IF(IPRINT.EQ.1) GO TO 431
   PRINT 109
   DO 42 INDEX=1,3
42 PRINT 111
   PRINT 110
   PRINT 14, (D(J),J=1,NC)
   PRINT 19, (CSAV(J),J=1,NC)
   PRINT 17, (DIRCOS(J),J=1,NC)
   PRINT 18, M1,DM1
   PRINT 109
   DO 43 INDEX=1,3
43 PRINT 111
   IF(IPRINT.EQ.3) PRINT 112

```

APPENDIX C - Continued

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C      CALL CNE-DIMENSIONAL SEARCH
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70700000

431 CALL CNEDIM
    DO 44 K=1,NC
    44 C(K) = CSAV(K)
        IF(IPRINT.NE.1) PRINT 905, I,WT,S,(C(K),K=1,NC)
        IF(ABS(S-C.).LT.1.E-8) GO TO 20
    50 CONTINUE

C      CENTRAL DIFFERENCE CALCULATION OF FIRST DERIVATIVES

        IF(IPRINT.NE.3) GO TO 203
        PRINT 15
        PRINT 5
        5 FORMAT(/* FLETCHER - POWELL SEARCH ROUTINE*/ * PERTURBED CASES*)
    203 CONTINUE
        KDERIV = 1
        DO 51 K=1,NC
        C      SET D2 = 0 FOR INACTIVE DESIGN VARIABLES
            IF(DC(K).EQ.0.) GO TO 52
            C(K) = CSAV(K)+C(K)
            CALL PENFUN(IPLLS)
            IF(IFAIL.EQ.1) GO TO 502
            IF(IPRINT.NE.3) GO TO 204
            PRINT 3, IPLLS,(C(J),J=1,NC)
            PRINT 15
        204 CONTINUE
            C(K) = CSAV(K)-C(K)
            CALL PENFUN(IMINUS)
            IF(IFAIL.EQ.1) GO TO 502
            IF(IPRINT.NE.3) GO TO 205
            PRINT 4, IMINUS,(C(J),J=1,NC)
            PRINT 15
        205 CONTINUE
            D2(K) = (IPLUS-IMINUS)/(2.*DC(K))+AD(K)
            C(K) = CSAV(K)
            GO TO 51
        52 CONTINUE
            D2(K) = 0.
        51 CONTINUE
            GO TO 60
        502 DO 512 J=1,NC
            C(J) = CSAV(J)

```

```

512 DC(J) = DC(J)/2.
    IFAIL = 0
    GN TO 50

C   GRADIENT CONVERGENCE TEST

60 SUM = 0.
    DO 61 K=1,NC
61 SUM = SUM+D2(K)*#2
    G = SORT(SUM)
    IF(G.GT..7*WT) GO TO 70

C   NOTE - VALUE USED IN GRADIENT CONVERGENCE TEST MAY BE MODIFIED
C   TO SUIT USER

    IF(IPRINT.NE.1) PRINT 62
    GO TO 300

C   UPDATE HH MATRIX

70 CONTINUE

C   REDUCE D, D2, AND H VECTORS TO NCA (ACTIVE DESIGN VARIABLES ONLY)

    NCA = NC
    IF(NC.EQ.1) GO TO 83
    K = NC-1
79 CONTINUE
    IF(DC(K).NE.0.) GO TO 77
    NCA = NCA-1
    DO 78 KK=K,NCA
    D(KK) = D(KK+1)
    D2(KK) = D2(KK+1)
    H(KK) = H(KK+1)
    DIRCOS(KK) = DIRCOS(KK+1)
78 CONTINUE
77 CONTINUE
    K = K-1
    IF(K.EQ.0) GO TO 82
    GO TO 79

82 CONTINUE
    IF(DC(NC).EQ.0.) NCA = NCA-1
83 CONTINUE

```

```

DO 71 K=1,NCA
  YT(K) = Y(K) - D2(K)-D(K)
71 SIG(K) = S*DIRCOS(K)
  CALL MATRIX(MULT,1,NCA,NCA,YT,1,HH,16,YT,1)
  CALL MATRIX(MULT,NCA,1,NCA,Y,16,YT,1,B,16)
  CALL MATRIX(MULT,NCA,NCA,NCA,HH,16,B,16,B,16)
  SCLR = 0.
DO 72 K=1,NCA
  SCLR = SCLR+YT(K)*Y(K)
72 SCLR = 1./SCLR
  CALL MATRIX(ISCLR,NCA,NCA,0,SCLR,0,B,15,B,16)
  CALL MATRIX(ISLB,NCA,NCA,0,HH,16,B,16,HH,16)
  SCLR = 0.
DO 73 K=1,NCA
  SIGT(K) = SIG(K)
73 SCLR = SCLR+SIGT(K)*Y(K)
  SCLR = 1./SCLR
  CALL MATRIX(MULT,NCA,1,NCA,SIG,16,SIGT,1,A,16)
  CALL MATRIX(ISCLR,NCA,NCA,0,SCLR,0,A,15,A,16)
  CALL MATRIX(IACC,NCA,NCA,0,HH,15,A,16,HH,16)
  DO 74 K=1,NCA
    D(K) = D2(K)
    IF(KCUNTR.GE.(NCA+3)) GO TO 129
    IF (KOUNT.LT.NCA) GO TO 30
74
C    QUADRATIC CONVERGENCE TEST
    CALL MATRIX(MULT,1,NCA,NCA,D2,1,HH,16,D2,1)
C    EXPAND 0 AND D2 VECTORS TO NC
    IF(NC.EQ.1) GO TO 96
    K1 = NC-1
    DO 94 K=1,K1
      IF(DC(K).NE.0.) GO TO 94
      J1 = NCA
      DO 95 KK=K,NCA
        D(J1+1) = C(J1)
        D2(J1+1) = D2(J1)
        J1 = J1-1
95 CONTINUE
      NCA = NCA+1
      D(K) = D2(K) = 0.

```

```

94 CONTINUE
  IF(DC(NC).EQ.0.) D(NC) = D2(NC) = 0.
96 CONTINUE
  Q = 0.
  DO 75 K=1,NC
    Q = Q+D2(K)*D(K)
  75 Q = 1./2.*Q/(WT+PF)
  IF(IPRINT.NE.1) PRINT 998, Q
998 FORMAT(/* Q =*E16.8)
  IF(Q.GT.0.) GO TO 80
  IF(IPRINT.NE.1) PRINT 76
  GO TO 20
80 IF(Q.GT..02) GO TO 90
  IF(IPRINT.NE.1) PRINT 81
  GO TO 300

C  MINIMUM MOVE SIZE CONVERGENCE TEST

90 SUM = 0.
  DO 91 K=1,NC
    91 SUM = SUM+D(K)**2
    SUM = SQRT(SUM)
    DCR = 0.
  DO 92 K=1,NC
    DIRCOS(K) = -D(K)/SUM
    IF(ABS(DIRCOS(K)).LT.DCR) GO TO 92
    KK = K
    DCR = ABS(DIRCOS(K))
92 CONTINUE
  DO 93 K=1,NC
    H(K) = 20.*DIRCOS(K)*DC(KK)
    C(K) = CSAV(K)+H(K)
93 CONTINUE
  KOERIV = 0
  CALL PENFUN(M2)
  IF(IFAIL.EQ.0) GO TO 150
100 IF(IPRINT.NE.1) PRINT 101
  IFAIL = 0
  GO TO 300
150 CONTINUE
  IF(IPRINT.NE.3) GO TO 206
  PRINT 6, M2, C(J), J=1,NC)
  6 FORMAT(/* M2 =*E16.8/(5E25.8))

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APPENDIX C - Continued

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PRINT 15
206 CONTINUE
DO 151 K=1,NC
151 C(K) = CSAV(K)+1.5*H(K)
KDERIV = 1
CALL PENFUN(MPLUS)
IF(IFAIL.EQ.0) GO TO 120
GO TO 100
120 SUM = 0.
IF(IPRINT.NE.3) GO TO 207
PRINT 7, MPLUS,(C(J),J=1,NC)
7 FORMAT(/* MPLUS =*E16.8/(5E25.8))
PRINT 15
207 CONTINUE
DO 121 K=1,NC
121 SUM = SUM+H(K)**2
DM2 = (MPLUS-M2)/(.5*SQR(T(SUM)))
DO 122 K=1,NC
122 DM2 = DM2+DIRCOS(K)*AD(K)
IF(DM2.LT.0.) GO TO 49
GO TO 100
129 IF(IPRINT.NE.1) PRINT 128, KGUNTR,R
128 FORMAT(/* NCA+3 =*I3* ONEDIMENSIONAL SEARCHES HAVE BEEN CARRIED CU
1T FOR R =*E16.8* - CONVERGENCE ASSUMED*/)
300 CTEST = PF/(WT+PF)
IF(IPRINT.NE.1) PRINT 921, CTEST
IF(CTEST.LT.0.) GO TO 20
IF(CTEST.GT.0.02) GO TO 310
PRINT 15
PRINT 306
306 FORMAT(* THE FOLLOWING INFORMATION IS FOR THE FINAL DESIGN*)
IF(IPRINT.EQ.1) PRINT 921, CTEST
PRINT 301
DO 305 J=1,NC
305 C(J)=CSAV(J)
CALL WEIGHT(FWT)
PRINT 309, FWT
309 FORMAT(//* FINAL WEIGHT=*E16.8/)
PRINT 308, (C(J),J=1,NC)
308 FORMAT(//* FINAL CONSTANTS*/(4E20.8))
NEWR = 1
CALL PENFUN(I)
CALL TABLE

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APPENDIX C - Continued

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310 RETURN
    R = R/RDC
    IF (IPRINT.NE.1) PRINT 311, R
    PF = PF/RDC
    IF (IPRINT.NE.1) PRINT 1, PF
    IF (IPRINT.NE.3) GO TO 10
    PRINT 15
    PRINT 5
    GO TO 10
END
```

APPENDIX C - Continued

```

SUBROUTINE ONEDIM
C  ONE-DIMENSIONAL SEARCH USING DIRECTION COSINES CALCULATED IN FPWEL

332 FORMAT(/* S =*E16.8/
1* B*E16.8/* O2,D3*2E16.8,5X*E2,E3*2E16.8,5X*F2,F3*2E16.8/
1* P =*E16.8,5X* PPRIME =*E16.8)
334 FORMAT(/* G1,G2,G3*3E16.8)
335 FORMAT(* S(*I2*) =*E16.8)
338 FORMAT(////* J =*I3)
800 FORMAT(/* V0,VP,VPP,*3E20.8/* OMEGA0,OMEGAP,OMEGAPP*3E20.8/
1* V1,OMEGA1*2E20.8)

EXTERNAL FQFS
LOGICAL DESCON
REAL MU
DIMENSION H(16)
DIMENSION A(3,5)
DIMENSION S(10),G1(10),G2(10),G3(10)
DIMENSION SA(10)
DIMENSION QMA(10),VA(10)
COMMON
2/BLK2/OMEGA,V
3/BLK3/ANG,RHOW,E,MU,C(16),SS,EPS
5/BLK5/OMEGA1,DELOM,OMFIN,V1,DELV,VFIN
1/BLK6/IOPT,IPRINT,ISNDWCH
2/BLK12/VCR,R,NC,H,DESCON(3)
7/BLK17/NEWR
2/BLK20/IFIRST,IFAIL,NOCONV,NOS
1/BLK28/CSAV(16),DC(16),D(16),DIRCOS(16),DM1,PF
3/BLK30/B,D2,D3,E2,E3,F2,F3,L,INEAR,VP,G,VPP
1/BLK31/DCR,OMPLUS,VPLUS
2/BLK32/IG
3/BLK33/GG1,GG2,GG3
4/BLK34/GP1,GP2,GP3
5/BLK35/OMEGA0,V0
7/BLK37/G11,G22,G33
4/BLK40/AD(16)
1/BLK41/WT
3/BLK43/SRETURN
IF(DESCON(1)) 200,201
201 CONTINUE

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VPLUS = OMPLUS = 2.
V = VO = OMEGA = OMEGAO = 2.
VPP = OMEGAPP = 0.
VCR = 1.
GLAPRX = 1.
200 CONTINUE
VP = (VPLUS-VO)/DCR
OMEGAP = (CMPLUS-OMEGAO)/CCR
OMA(1) = OMEGAO
VA(1) = VO
EP1 = 1.E-3
EP2 = 1.E-4
MAXI = 100
J = 1
IF(IPRINT.EQ.3) PRINT 338, J
S(1) = 0.
SA(1) = 0.

C LINEAR APPROXIMATION TO G AND F USED FOR PPRIME

AP = 0.
B = 0.
DO 100 I=1,NC
AP = AP+CSAV(I)*AD(I)
B = B+AD(I)*DIRCOS(I)
100 CONTINUE
D2 = G2(1) = 1.-G22
D3 = G3(1) = 1.-G33
E2 = (G22-GP2) /DCR
E3 = (G33-GP3) /DCR
F1 = 0.
V1 = VO+S(1)*VP
G1(1) = 1.-VCR/V1
PPRIME = B-R*(VCR*VP/(V1**2*G1(1)**2)+30.*E2/(D2+E2*S(1))**2
1+30.*E3/(D3+E3*S(1))**2)
P = AP+B*S(1)+R*(1./G1(1)+30./(D2+E2*S(1))+30./(D3+E3*S(1)))
IF(IPRINT.EQ.3) PRINT 332, S(1),B,D2,D3,E2,E3,F2,F3,P,PPRIME
IF(PPRIME.LE.0.) GO TO 110
PRINT 10
10 FORMAT(///* PPRIME POSITIVE AT S = 0*//)
STOP
110 CONTINUE

```

APPENDIX C - Continued

```

C LOCATE S(2) USING LINEAR APPROXIMATION
  J = 2
  SI = S(1)
  SF = .2
  SDEL = 20.*DCR
  LINEAR = 1
  G = G1(1)
  CALL ITR2(S(2),SI,SF,SDEL,FOFS,EP1,EP2,MAXI,ICODE)
  IF(ICODE.EQ.0) GO TO 5
  IF(ICODE.NE.3) GO TO 1000
  S(2) = .08
  GO TO 76
  5 S(2) = .4*S(2)
  76 CONTINUE
  IF(IPRINT.EQ.3) PRINT 338, J
  77 CONTINUE
  V1 = VC+S(2)*VP
  OMEGAL = OMEGA0+S(2)*OMEGAP
  1 CONTINUE
  DO 78 I=1,NC
  C(I) = CSAV(I)+S(2)*DIRCOS(I)
  78 CONTINUE
  IF(DESCON(1)) 202,2
  202 CONTINUE
  IG = 1
  CALL PENFUN(X)
  IF(IFAIL.EQ.0) GO TO 79
  IFAIL = 0
  S(2) = .5*S(2)
  GO TO 77
  79 CONTINUE
  VA(2) = V
  OMA(2) = OMEGA
  SA(2) = S(2)
  VPP = (V-V0-S(2)*VP)/S(2)**2
  OMEGAPP = (OMEGA-OMEGA0-S(2)*OMEGAP)/S(2)**2
  80 CONTINUE
  V1 = V0+S(2)*VP+S(2)**2*VPP
  G1APRX = 1.-VCR/V1
  IF(G1APRX.GT.0.) GO TO 2
  S(2) = .8*S(2)
  V1 = V0+S(2)*VP+S(2)**2*VPP

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APPENDIX C - Continued

```

OMEGA1 = CMEGAO+S(2)*OMEGAP+S(2)**2*OMEGAPP
GO TO 1
2 CONTINUE
IG = 2
IF(IPRINT.EQ.3) PRINT 80C, VO,VP,VPP,OMEGAO,CMEGAP,OMEGAPP,V1,
1OMEGA1
CALL PENFUN(X)
IF(IFAIL.EQ.C) GO TO 6
IFAIL = 0
S(2) = .5*S(2)
DC 81 I=1,NC
C(I) = CSAV(I)+S(2)*DIRCOS(I)
81 CONTINUE
IF(DESCCN(1)) 2C3,2
203 CONTINUE
GC TC 80
6 CONTINUE
G1(2) = G1APRX
G2(2) = 1.-GC2
G3(2) = 1.-GG3

C CALCULATE PARABOLIC APPROXIMATION TO G2 AND G3 USING VALUES AT S(2)
F2 = (G2(2)-C2-E2*S(2))/S(2)**2
F3 = (G3(2)-C3-E3*S(2))/S(2)**2
3 CONTINUE
PPRIME = B-R*(VCR*(VP+2.*S(2)*VPP)/(VC+S(2)*VP+S(2)**2*VPP-VCR)**2
1+(E2+2.*F2*S(2))/(D2+E2*S(2)+F2*S(2)**2)**2**2*30.
2+(E3+2.*F3*S(2))/(D3+E3*S(2)+F3*S(2)**2)**2**2*30.)
P = AP+B*S(2)+R*(1./G1(2)
1+30./(D2+E2*S(2)+F2*S(2)**2)+30./(D3+E3*S(2)+F3*S(2)**2))
IF(IPRINT.EQ.3) PRINT 332, S(2),B,D2,D3,E2,E3,F2,F3,P,PPRIME
4 CCNTINUE

C LOCATE S(3) USING PARABOLIC APPROXIMATION
J = 2
SI = S(2)
SF = S(2)+PPRIME/(-ABS(PPRIME))**2.*S(2)
IF(SI.LT.SF) GC TO 15
X = SI
SI = SF
SF = X

```

APPENDIX C - Continued

```

15 CONTINUE
SDEL = S(2)/10.
LINEAR = 0
G = G1(2)
CALL ITR2(S(3),SI,SF,SDEL,FOFS,EPI,EP2,MAXI,ICODE)
IF(ICODE.EQ.0) GO TO 18
IF(ICODE.NE.3) GO TO 1000
S(3) = SF
IF(PRIME.GT.0.) S(3) = SI
GO TO 11
18 S(3) = S(2)+.6*(S(3)-S(2))
11 CONTINUE
IF(IPRINT.EQ.3) PRINT 338, J
DO 88 I=1,NC
C(I) = CSAV(I)+S(3)*DIRCOS(I)
88 CONTINUE
IF(DESCCN(1)) 212,12
212 CONTINUE
V1 = VO+S(3)*VP+S(3)**2*VPP
OMEGA1 = OMEGA0+S(3)*OMEGAP+S(3)**2*CMEGAPP
IG = 1
CALL PENFUN(X)
IF(IFAIL.EQ.0) GO TO 89
IFAIL = 0
S(3) = S(2)+.8*(S(3)-S(2))
GO TO 11
89 CONTINUE
SA(3) = S(3)
VA(3) = V
OMA(3) = OMEGA
VPP = ((VA(2)-VO)/SA(2)-(VA(3)-VO)/SA(3))/(SA(2)-SA(3))
VP = (VA(3)-VO)/SA(3)-VPP*SA(3)
OMEGAPP = ((OMA(2)-OMEGA0)/SA(2)-(OMA(3)-OMEGA0)/SA(3))/(SA(2)-SA(
13))
OMEGAP = (OMA(3)-OMEGA0)/SA(3)-CMEGAPP*SA(3)
90 CONTINUE
V1 = VO+S(3)*VP+S(3)**2*VPP
G1APRX = 1.-VCR/V1
IF(G1APRX.GT.0.) GO TO 12
S(3) = S(2)+.8*(S(3)-S(2))
GO TO 11
12 CONTINUE
IG = 2

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106100000
106200000
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110000000
110100000
110200000
110300000

APPENDIX C - Continued

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110400000
110500000
110600000
110700000
110800000
110900000
111000000
111100000
111200000
111300000
111400000
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114000000
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114200000
114300000
114400000
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114600000

CALL PENFUN(X)
IF(IFAIL.EQ.0) GO TO 121
IFAIL = 0
S(3) = S(2)+.6*(S(3)-S(2))
DO 91 I=1,NC
  C(I) = CSAV(I)+S(3)*DIRCOS(I)
91 CONTINUE
  IF(DESCCN(1)) 213,12
213 CONTINUE
  GO TO 50
121 CONTINUE
  G1(3) = G1APRX
  G2(3) = 1.-GG2
  G3(3) = 1.-GG3

C   CALCULATE PARABOLIC APPROXIMATION TO G2 AND G3 USING VALUES AT S(3)
      F2 = ((G2(3)-D2)/S(3)-(G2(2)-D2)/S(2))/(S(3)-S(2))
      E2 = (G2(3)-D2)/S(3)-F2*S(3)
      F3 = ((G3(3)-D3)/S(3)-(G3(2)-D3)/S(2))/(S(3)-S(2))
      E3 = (G3(3)-D3)/S(3)-F3*S(3)
13 CONTINUE
  PPRIME = B-R*(VCR*(VP+2.*S(3)*VPP)/(VO+S(3)*VP+S(3)**2*VPP-VCR)**2
    1+(E2+2.*F2*S(3))/(D2+E2*S(3)+F2*S(3)**2)**2*30.
    2+(E3+2.*F3*S(3))/(D3+E3*S(3)+F3*S(3)**2)**2*30.)
  P = AP+B*S(3)+R*(1./G1(3)
    1+30./(D2+E2*S(3)+F2*S(3)**2)+30./(D3+E3*S(3)+F3*S(3)**2))
  IF(IPRINT.EQ.3) PRINT 332, S(3),B,D2,D3,E2,E3,F2,F3,P,PPRIME
14 CONTINUE

C   LOCATE S(4) USING PARABOLIC APPROXIMATION
      SI = S(3)
      SF = S(3)+PPRIME/(-ABS(PPRIME))*30.*ABS(S(3)-S(2))
      IF(SI.LT.SF) GO TO 16
      X = SI
      SI = SF
      SF = X
16 CONTINUE
      SOEL = ABS(S(3)-S(2))/10.
      G = G1(3)
      CALL ITR2(S(4),SI,SF,SOEL,FOFS,E1,EP2,MAXI,ICODE)
      IF(ICODE.EQ.0) GO TO 20

```


APPENDIX C - Continued

```

IF(ICODE.NE.3) GO TO 1000
S(4) = SF
IF(PPRIME.GT.0.) S(4) = SI
20 CONTINUE
J = 4
21 CONTINUE
IF(IPRINT.EQ.3) PRINT 338, J
DO 98 I=1,NC
C(I) = CSAV(I)+S(J)*DIRCCS(I)
98 CONTINUE
IF(DESCON(1)) 222,22
222 CONTINUE
V1 = VC+S(J)*VP+S(J)**2*VPP
OMEGA1 = OMEGA0+S(J)*OMEGAP+S(J)**2*OMEGAPP
IF(IPRINT.EQ.3) PRINT 800, VO,VP,VPP,OMEGA0,OMEGAP,OMEGAPP,V1,
1OMEGA1
IG = 1
CALL PENFUN(X)
IF(IFAIL.EC.0) GO TO 99
IFAIL = 0
S(J) = S(J-1)+.8*(S(J)-S(J-1))
GO TO 21
99 CONTINUE
SA(J) = S(J)
VA(J) = V
OMA(J) = OMEGA
KOUNT = 0

C CALCULATE PARABOLIC APPROXIMATION TO V AND OMEGA USING SA(J) AND 2 CLOSEST
C VALUES OF SA
IF(IPRINT.NE.3) GO TO 50
PRINT 320
330 FORMAT(/# S FOR PARABOLIC APPROXIMATION TO FLUTTER CONSTRAINT*)
PRINT 335, (I,SA(I),I=1,J)
50 CONTINUE
SAVE = SMINI1 = ABS(SA(J)-SA(1))
SMINI2 = ABS(SA(J)-SA(2))
IND1 = 1
IND2 = 2
IF(SMINI2.GE.SMINI1) GO TO 620
SMINI1 = SMINI2
SMINI2 = SAVE

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118900000

APPENDIX C - Continued

```

IND1 = 2
IND2 = 1
620 CONTINUE
J1 = J-1
DO 660 I=3,J1
SAVE = ABS(SA(J)-SA(I))
IF(SAVE.GT.SMINI2) GO TO 660
SMINI2 = SAVE
IND2 = I
IF(SMINI1.LT.SMINI2) GO TO 660
SAVE = SMINI2
SMINI2 = SMINI1
SMINI1 = SAVE
IND2 = IND1
IND1 = I
660 CONTINUE
A(1,1) = A(2,1) = A(3,1) = 1.
A(1,2) = SA(J)
A(1,3) = SA(J)**2
A(2,2) = SA(IND1)
A(2,3) = SA(IND1)**2
A(3,2) = SA(IND2)
A(3,3) = SA(IND2)**2
A(1,4) = VA(J)
A(2,4) = VA(IND1)
A(3,4) = VA(IND2)
A(1,5) = OMA(J)
A(2,5) = OMA(IND1)
A(3,5) = OMA(IND2)
ISOL = 10
CALL MATRIX(ISOL,3,5,0,A,3,DET)
VO = A(1,4)
VP = A(2,4)
VPP = A(3,4)
OMEGAO = A(1,5)
OMEGAP = A(2,5)
OMEGAPP = A(3,5)
101 CONTINUE
V1 = VO+S(J)*VP+S(J)**2*VPP
GLAPRX = 1.-VCR/V1
IF(GLAPRX.GT.0.) GO TO 22
S(J) = S(J-1)+.8*(S(J)-S(J-1))
GO TO 21

```

APPENDIX C - Continued

```

22 CONTINUE
IG = 2
CALL PENFUN(X)
IF(IFAIL.EQ.0) GO TO 122
IFAIL = 0
S(J) = S(J-1)+.8*(S(J)-S(J-1))
DO 102 I=1,NC
C(I) = CSAV(I)+S(J)*DIRCOS(I)
102 CONTINUE
KOUNT = 1
IF(DESCON(1)) 223,22
223 CONTINUE
GO TO 101
122 CONTINUE
G1(J) = G1APRX
G2(J) = 1.-GG2
G3(J) = 1.-GG3

C CALCULATE PARABOLIC APPROXIMATION TO G2 AND G3 USING S(J) AND TWO CLOSEST
C VALUES OF S
C IF(KOUNT.EQ.0.AND.DESCON(1)) GO TO 651
IF(IPRINT.NE.3) GO TO 51
PRINT 331
331 FORMAT(//* S FOR PARABOLIC APPROXIMATION TO STRESS AND MINGAGE CGN
1 STRAINTS*)
PRINT 335, (I,S(I),I=1,J)
51 CONTINUE
SAVE = SMINI1 = ABS(S(J)-S(1))
SMINI2 = ABS(S(J)-S(2))
IND1 = 1
IND2 = 2
IF(SMINI2.GE.SMINI1) GO TO 610
SMINI1 = SMINI2
SMINI2 = SAVE
IND1 = 2
IND2 = 1
610 CONTINUE
J1 = J-1
DO 650 I=3,J1
SAVE = ABS(S(J)-S(I))
IF(SAVE.GT.SMINI2) GO TO 650
SMINI2 = SAVE

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1234000000
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1240000000
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1270000000
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1272000000
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1275000000

APPENDIX C - Continued

```

IND2 = I
IF(SMINI1.LT.SMINI2) GO TO 650
SAVE = SMINI2
SMINI2 = SMINI1
SMINI1 = SAVE
IND2 = IND1
IND1 = I
650 CONTINUE
651 CONTINUE
A(1,1) = A(2,1) = A(3,1) = 1.
A(1,2) = S(J)
A(1,3) = S(J)**2
A(2,2) = S(IND1)
A(2,3) = S(IND1)**2
A(3,2) = S(IND2)
A(3,3) = S(IND2)**2
A(1,4) = G2(J)
A(2,4) = G2(IND1)
A(3,4) = G2(IND2)
A(1,5) = G3(J)
A(2,5) = G3(IND1)
A(3,5) = G3(IND2)
ISOL = 10
CALL MATRIX(ISOL,3,5,0,A,3,DET)
D2 = A(1,4)
E2 = A(2,4)
F2 = A(3,4)
D3 = A(1,5)
E3 = A(2,5)
F3 = A(3,5)
23 CONTINUE
PPRIME = B-R*(VCR*(VP+2.*S(J)*VPP)/(VO+S(J)*VP+S(J)**2*VPP-VCR)**2
1+(E2+2.*F2*S(J))/(D2+E2*S(J)+F2*S(J)**2)**2*30.
2+(E3+2.*F3*S(J))/(D3+E3*S(J)+F3*S(J)**2)**2*30.)
P = AP+B*S(J)+R*(1./G1(J)
1+30./(D2+E2*S(J)+F2*S(J)**2)+30./(D3+E3*S(J)+F3*S(J)**2))
IF(IPRINT.EQ.3) PRINT 332, S(J),B,D2,D3,E2,E3,F2,F3,P,PPRIME
IF(ABS(PPRIME).GT..2*ABS(CM1)) GO TO 24
C SLOPE CONVERGENCE TEST AT S(J)
IF(KOUNT.EQ.0) GO TO 527
IF(DESCON(1)) 227,527

```

APPENDIX C - Continued

```

227 CONTINUE
   KOUNT = 0
520 CONTINUE
   V1 = VO+S(J)*VP+S(J)**2*VPP
   OMEGAI = CMEGAO+S(J)*OMEGAP+S(J)**2*CMEGAPP
   IG = 1
   CALL PENFUN(X)
   G1(J) = 1.-GG1
   IF(IPRINT.EQ.3) PRINT 334, G1(J),G2(J),G3(J)
   IF(IFAIL.EQ.0) GO TO 521
   IFAIL = 0
   S(J) = S(J-1)+.8*(S(J)-S(J-1))
   DO 301 I=1,NC
   C(I) = CSAV(I)+S(J)*DIRCOS(I)
301 CONTINUE
   KOUNT = 1
   GO TO 520
521 CONTINUE
   IF(KOUNT)525,525,522

522 CONTINUE
   GIAPRX = G1(J)
   GO TO 23
525 CONTINUE
   IF(ABS(G1(J)-GIAPRX).GT..02) GO TO 526
527 CONTINUE
   IG = 0
   NEWR = 1
   IF(IPRINT.EQ.1) NEWR=0
   CALL WEIGHT(WT)
   OMEGAI = OMEGAO + S(J) * CMEGAP + S(J)**2 * CMEGAPP
   V1 = VO + S(J) * VP + S(J)**2 * VPP
   CALL PENFUN(PF)
   IF(IFAIL.EQ.0) GO TO 524
   IFAIL = 0
   PRINT 340
340 FORMAT(/* A FAILURE OCCURRED AFTER THE SLOPE CONVERGENCE TEST IN S
1UBROUTINE ONEDIM WAS SATISFIED.*/ * CCONSTRAINTS DO NOT APPEAR TO BE
2 UNIMODAL.*/)
   GO TO 21
524 CONTINUE
   OMEGAI=OMEGAO=OMEGA

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1319000000
1320000000
1321000000
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1350000000
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1358000000
1359000000
1360000000
1361000000

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136200000
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137000000
137100000
137200000
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139100000
139200000
139300000
139400000
139500000
139600000
139700000
139800000
139900000
140000000
140100000
140200000
140300000
140400000

V1=V0=V
SRETURN = S(J)
G11 = GG1
G22 = GG2
G33 = GG3
NEW = 0
DO 523 I=1,NC
  CSAV(I) = CSAV(I)+S(J)*DIRCOS(I)
523 CONTINUE
  RETURN
526 CONTINUE
  IF(KOUNT.EQ.0) GO TO 24
  PPRIME = B-R*(VCR*(VP+2.*S(J)*VPP)/(V0+S(J)*VP+S(J)**2*VPP-VCR)**2
  1+(E2+2.*F2*S(J))/(D2+E2*S(J)+F2*S(J)**2)**2*30.
  2+(E3+2.*F3*S(J))/(D3+E3*S(J)+F3*S(J)**2)**2*30.)
  P = AP+B*S(J)+R*(1./G1(J)
  1+30./(D2+E2*S(J)+F2*S(J)**2)+30./(D3+E3*S(J)+F3*S(J)**2))
  IF(IPRINT.EQ.3) PRINT 332, S(J), B, D2, D3, E2, E3, F2, F3, P, PPRIME
24 CONTINUE

C LOCATE S(J+1) USING PARABOLIC APPROXIMATION

SI = S(J)
SF = S(J)+PPRIME/(-ABS(PPRIME))*30.*ABS(S(J)-S(J-1))
IF(SI.LT.SF) GO TO 17
X = SI
SI = SF
SF = X
17 CONTINUE
SDEL = ABS(S(J)-S(J-1))/10.
G = G1(J)
CALL ITR2(S(J+1),SI,SF,SDEL,FOFS,EP1,EP2,MAXI,ICODE)

IF(ICODE.EQ.0) GO TO 31
IF(ICODE.NE.3) GO TO 1000
S(J+1) = SF
IF(PPRIME.GT.0.) S(J+1) = SI
31 CONTINUE
J = J+1
IF(J.LT.11) GO TO 21
PRINT 1002
1002 FORMAT(/* ONEDIM J=10 STOP*)
STOP

```

APPENDIX C - Continued

1405000000
1406000000
1407000000
1408000000

1000 PRINT 1001, ICODE
1001 FORMAT (// * ERROR RETURN FROM ITR2 ICODE =*I2)
STOP
END

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1409000000
1410000000
1411000000
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1413000000
1414000000
1415000000
1416000000
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1419000000
1420000000
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1429000000

FUNCTION FOFS(S)
C   CALLED BY ITR2 IN ONEDIM TO APPROXIMATE P FUNCTION

LOGICAL DESCN
DIMENSION H(16)
COMMON
2/BLK12/VCR,R,NC,H,DESCN(3)
3/BLK30/B,D2,D3,E2,E3,F2,F3,L,VP,G,VPP
5/BLK35/CMEGAO,V0
IF(L.EQ.1) GO TO 100
FOFS = B-R*(VCR*(VP+2.*S*VPP)/(V0+S*VP+S*S*VPP-VCR)**2
1+30.*(E2+2.*F2*S)/(D2+E2*S+F2*S**2)**2
2+30.*(E3+2.*F3*S)/(D3+E3*S+F3*S**2)**2)
RETURN
100 CONTINUE
FOFS = B-R*(VCR*VP/(V0+S*VP-VCR)**2
1+30.*E2/(D2+E2*S)**2+30.*E3/(D3+E3
1*S)**2)
RETURN
END

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APPENDIX C – Continued

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143000000
143100000
143200000
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143700000
143800000
143900000
144000000

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SUBROUTINE WEIGHT(WT)
REAL MU
COMMON
3/BLK3/ANG,RHOW,E,MU,C(16),SS,EPS
3/BLK40/AD(16)
1/BLK12/VCR,R,NC
WT = 0.
DO 100 I=1,NC
100 WT = WT+C(I)*AD(I)
RETURN
END

```

```

SUBROUTINE PENFUN(PF)
C  CALCULATES PENALTY FUNCTION

LOGICAL DESCON
REAL MU
DIMENSION H(16)
COMMON
2/BLK2/CMEGA,V
3/BLK3/ANG,RHOW,E,MU,C(16),SS,EPS
1/BLK6/IOPT,IPRINT,ISNDWCH
8/BLK8/KDERIV
1/BLK12/VCR,R,NC,H,DESCON(3)
7/BLK17/NEWR
1/BLK20/IFIRST,IFAIL,NOCONV,NOS
2/BLK32/IG
3/BLK33/G1,G2,G3

C  BEGIN FLUTTER CONSTRAINT. FLUTTER VELOCITY V MUST BE GREATER THAN VCR,
C  THE CRITICAL FLUTTER VELOCITY.

IF(IG.EQ.2) GO TO 60
IF(DESCCN(1)) GO TO 50
G1 = 0.
GO TO 60
50 CONTINUE
NOCONV = 0
CALL FLUTTER
IF(IFIRST.EQ.1.AND.V.LT.VCR) PRINT 666
666 FORMAT(* V.LT. VCR - STOP*)
IF(IFIRST.EQ.1.AND.V.LT.VCR) STOP
IF(IPRINT.EQ.1) GO TO 10
IF(IFIRST.EQ.2.AND.NOCONV.EQ.1) PRINT 667
667 FORMAT(* NOCONV - RETURN*)
10 IF(IFIRST.EQ.2.AND.NOCONV.EQ.1) GO TO 100
IF(IPRINT.EQ.1) GO TO 20
IF(IFIRST.EQ.2.AND.V.LT.VCR) PRINT 668
668 FORMAT(* V.LT.VCR - REDUCE S OR DC*)
20 IF(IFIRST.EQ.2.AND.V.LT.VCR) GC TO 100
GO TO 400
100 CONTINUE
IFAIL=1

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144200000
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147000000
147100000
147200000
147300000
147400000
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147600000
147700000
147800000
147900000
148000000
148100000
148200000

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RETURN
400 CONTINUE
    G1 = VCR/V

C BEGIN STRESS CONSTRAINT. STRESS CONDITION Y AT EACH POINT ON THE WING
C SURFACE MUST BE LESS THAN YCR, THE CRITICAL STRESS CONDITION.

60 CONTINUE
    IF(IG.EQ.1) GO TO 777
    IF(DESCON(2)) GO TO 61
    G2 = 0.
    GO TO 70
61 CONTINUE
    IFAIL=0
    CALL STRESS(G2)
    IF(IFAIL.EQ.1) RETURN

C BEGIN MINIMUM GAGE CONSTRAINT. TBAR IS THE DEPTH OF THE WING IF THE WING
C IS SOLID. TBAR IS THE THICKNESS OF THE UPPER OR LOWER FACE IF THE WING
C IS A SANDWICH. TBAR MUST BE GREATER THAN TBARMIN, THE MINIMUM THICKNESS
C ALLOWED.

70 IF(DESCON(3)) GO TO 80
    G3 = 0.
    GO TO 777
80 CONTINUE
    CALL MINGAGE(G3)
    IF(IFAIL.EQ.1) RETURN
777 CONTINUE

C IN THIS PROGRAM G1, G2, AND G3 ARE THE RATIOS OF THE BEHAVIOR
C VARIABLES FLUTTER, STRESS, AND MINIMUM GAGE TO THE CRITICAL VALUES
C OF THESE VARIABLES. ALL COMPUTATIONS CARRIED OUT IN THE PROGRAM
C USE THAT DEFINITION. FOR EXAMPLE, G1 = VCR/V. THE QUANTITIES
C CALLED G1, G2, G3 IN THE OUTPUT AGREE WITH CONVENTIONAL NOTATION.
C FOR EXAMPLE, G1(OUTPUT) = 1-G1(INTERNAL).
C ALSO, FAILURE RATIO 1(OUTPUT) = G1(INTERNAL).

    GSUB1=1.-G1
    GSUB2=1.-G2
    GSUB3=1.-G3
    IF(1PRINT.EQ.3.OR.NEWR.EQ.1) PRINT 1, G1,GSUB1,G2,GSUB2,G3,GSUB3
1 FORMAT(/# FAILURE RATIO 1 =*E16.8,5X*G1 =*E16.8/* FAILURE RATIO 2

```

APPENDIX C - Continued

152600000
152700000
152800000
152900000
153000000
153100000

```

1=*E16.8,5X*G2=*E16.8/* FAILURE RATIO 3=*E16.8,5X*G3=*E16.8)
SUM = 1./((1.-G1)+1./((1.-G2)*30.+1./((1.-G3)*30.
PF = R*SUM
NEWIR = G
RETURN
END

```

APPENDIX C - Continued

```

SUBROUTINE STRESS(G2)
C THIS SUBROUTINE SOLVES FOR DEFLECTIONS CAUSED BY A STATIC LOAD ON THE
C SURFACE OF THE WING. THE DEFLECTION IS IN TERMS OF PHI. THE LOADING
C IS A UNIFORM PRESSURE LOAD DENOTED P OF INTENSITY PLOAD (PSI).

LOGICAL DESCN
EXTERNAL FUNCS
EXTERNAL FSTRESS
REAL MU
COMPLEX AAC
DIMENSION H(16)
DIMENSION EPS4P(33,1),P(11,1),
LAST(33,33),PIVOT(33),
ISX(11),FOFX(1),ANS(1)
COMMON
1/BLK1/NN,NN1,NN2,NN22,NN23,NN33,MCS,NCS,NMAX
3/BLK3/ANG,RHOW,E,MU,C(16),SS,EPS
1/BLK6/IOPT,IPRINT,ISNDWCH
1/BLK11/EPS4P
1/BLK20/IFIRST,IFAIL,NOCONV,NQS
1/BLK13/Y,C1,C2
2/BLK14/YCR,IY
1/BLK15/PLOAD
7/BLK17/NEW
1/BLK12/VCR,R,NC,H,DESCN(3)
2/BLK32/IG
1/NEWBLK/AAC(33,33),ASTRESS(33,33),S(5,11),F(5,11),HP(5,11)
1/BLKNPS/AREA
NAMELIST/STRSS/PLOAD,YCR
NMAX=33
ISCLR = 24

C LOADING IS A COLUMN VECTOR CALLED P
IF(IPRINT.EQ.0) GO TO 151
IF(IG.EQ.2) GO TO 152
IF(DESCCN(1)) GO TO 151
152 CONTINUE
CALL AAMAT
CALL SMAT
151 CONTINUE

```

```

153200000
153300000
153400000
153500000
153600000
153700000
153800000
153900000
154000000
154100000
154200000
154300000
154400000
154500000
154600000
154700000
154800000
154900000
155000000
155100000
155200000
155300000
155400000
155500000
155600000
155700000
155800000
155900000
156000000
156100000
156200000
156300000
156400000
156500000
156600000
156700000
156800000
156900000
157000000
157100000
157200000
157300000

```

APPENDIX C - Continued

```

IF(IPRINT.NE.0) GO TO 150
READ STRSS
PRINT 10
PRINT 11, ((ASTRESS(I,J),J=1,NN1),I=1,NN33)
PRINT 13
PRINT 11, ((ASTRESS(I,J),J=NN2,NN22),I=1,NN33)
PRINT 13
PRINT 11, ((ASTRESS(I,J),J=NN23,NN33),I=1,NN33)
10 FORMAT(1H1////* ASTRESS MATRIX (ORIGINAL AA MATRIX USED TO BUILD S
1TRESS MATRIX)*//)
11 FORMAT(5E16.8)
13 FORMAT(1H1////////)
150 CONTINUE

C CALCULATE P BY ANALYTIC INTEGRATION, MULTIPLY BY EPS**4,
C AND STORE IN EPS4P
EPS4=EPS**4
DO 200 I=1,NN1
Y=EPS*FLOAT(I)
CALL CEE
P(I,1)=PLOAD*(C2-C1)
IF(I.EQ.NN1-1) P(I,1)=P(I,1)/2.
IF(I.EQ.NN1) P(I,1)=0.
EPS4P(I,1)=EPS4*P(I,1)
II=I+NN1
P(I,1)=PLOAD*(C2**2/2.-C1**2/2.)
IF(I.EQ.NN1-1) P(I,1)=P(I,1)/2.
IF(I.EQ.NN1) P(I,1)=0.
EPS4P(II,1)=EPS4*P(I,1)
III=I+NN2
P(I,1)=PLOAD*(C2**3/3.-C1**3/3.)
IF(I.EQ.NN1-1) P(I,1)=P(I,1)/2.
IF(I.EQ.NN1) P(I,1)=0.
200 EPS4P(III,1)=EPS4*P(I,1)
IORD=NN33
DO 600 I=1,IORD
DO 600 J=1,IORD
600 AST(I,J) = ASTRESS(I,J)
IF(IPRINT.NE.0) GO TO 55
PRINT 15, PLOAD
15 FORMAT(////* PLOAD =*E16.8//)
PRINT 16, (EPS4P(I,1),I=1,IORD)

```

APPENDIX C - Continued

```

16 FORMAT(///6X*EPS4P*/(E16.8))
55 CONTINUE

C SOLVE FOR PHI AND STORE IN EPS4P

CALL SIMEQ(AST,IORD,EPS4P,1,DETERM,PIVOT,NMAX,ISCALE)
IF(IPRINT.NE.0) GO TO 250
PRINT 251, (EPS4P(I,1),I=1,IORD)
251 FORMAT(///7X*PHI*/(E16.8))
RETURN
250 CONTINUE
DO 500 I=1,NN1
  IY = I
  Y = EPS*FLOAT(I-1)
  IF(NEWR.EQ.1.AND.I.EQ.1) PRINT 651
  651 FORMAT(// * STRESS TABLE*//)
  IF(NEWR.EQ.1) PRINT 650, Y
  650 FORMAT(// * Y =E16.8/4X*X*,13X*-ZU*,11X*EP X*,10X*EP Y*,10X*GAM XY*
    18X*SIG X*,9X*SIG Y*,9X*TAU XY*,8X*Y*)
  CALL CEE
  CALL MGAUSS(C1,C2,NCS,ANS,FSTRESS,FOFX,1)
  IF(IFAIL.EQ.1) RETURN
  500 SX(I) = ANS(1)
  SUM = 1./2.*SX(1)+1./2.*SX(NN1)
  DO 300 I=2,NN
    300 SUM = SUM+SX(I)
  SUM = SUM*EPS/AREA
  G2 = 1.-1./SUM
  RETURN
END
1617000000
1618000000
1619000000
1620000000
1621000000
1622000000
1623000000
1624000000
1625000000
1626000000
1627000000
1628000000
1629000000
1630000000
1631000000
1632000000
1633000000
1634000000
1635000000
1636000000
1637000000
1638000000
1639000000
1640000000
1641000000
1642000000
1643000000
1644000000
1645000000
1646000000

```

```

SUBROUTINE FSTRESS(X,FOFX)
C THIS SUBROUTINE TAKES THE STRAINS CALCULATED IN SUBROUTINE EGAMMA AND
C CALCULATES THE STRESSES.

REAL MU
DIMENSION FOFX(1)
COMMON
3/BLK3/ANG,RHOW,E,MU,C(16),SS,EPS
1/BLK6/IOPT,IPRINT,ISNDWCH
2/BLK14/YCR,IY
7/BLK17/NEWR
1/BLK20/IFIRST,IFAIL,NOCONV,NOS
CALL EGAMMA(EX,EY,GAMXY,X,IY,ZU)
SIGX = E/(1.-MU**2)*(EX+MU*EY)
SIGY = E/(1.-MU**2)*(EY+MU*EX)
TAUXY = E/(2.*(1.+MU))*GAMXY
Y = SQRT(SIGX**2-SIGX*SIGY+SIGY**2+3.*TAUXY**2)
IF(NEWR.EQ.1) PRINT 2, X,ZU,EX,EY,GAMXY,SIGX,SIGY,TAUXY,Y
2 FORMAT(9E14.5)
IF(Y.LE.YCR) GO TO 200
IF(IFIRST.EQ.1) PRINT 10
10 FORMAT(/* INITIAL DESIGN UNACCEPTABLE BECAUSE YIELD STRESS EXCEEDS
1D*)
IF(IFIRST.EQ.1) STOP
IFAIL=1
IF(IPRINT.NE.1) PRINT 11
11 FORMAT(/* YIELD STRESS EXCEEDED - REDUCE S OR DC*)
RETURN
200 CONTINUE
FOFX(1) = 1./(1.-Y/YCR)
RETURN
END
164700000
164800000
164900000
165000000
165100000
165200000
165300000
165400000
165500000
165600000
165700000
165800000
165900000
166000000
166100000
166200000
166300000
166400000
166500000
166600000
166700000
166800000
166900000
167000000
167100000
167200000
167300000
167400000
167500000
167600000
167700000
167800000
167900000

```


APPENDIX C - Continued

```

SUBROUTINE EGAMMA(EX,EY,GAMXY,X,I,ZU)
C  CALCULATES STRAINS

REAL MU
DIMENSION PHI(22)
COMMON
1/BLK1/NN,NN1,NN2,NN22,NN23,NN33,MCS,NCS,NMAX
3/BLK3/ANG,RHOW,E,MU,C(16),SS,EPS
3/BLK11/PHI
CALL DEPTH(H,X)
ZU = -H/2.
I1 = I+NN1
I2 = I+2*NN1
EX = 2.*ZU*PHI(I2-1)
IF(I.EQ.1) EX = 0.
IF(I-2)100,101,102
100 EY = 2.*ZU/EPS**2*(PHI(I)+X*PHI(I1)+X**2*PHI(I2))
    GAMXY = 0.
    RETURN
101 EY = ZU/EPS**2*(-2.*PHI(I-1)+PHI(I)+X*(-2.*PHI(I1-1)+PHI(I1))+
    1X**2*(-2.*PHI(I2-1)+PHI(I2)))
    GAMXY = ZU/EPS*(PHI(I1)+2.*X*PHI(I2))
    RETURN
102 EY = ZU/EPS**2*(PHI(I-2)-2.*PHI(I-1)+PHI(I)+X*(PHI(I1-2)-2.*PHI(I1
    1-1))+PHI(I1))+X**2*(PHI(I2-2)-2.*PHI(I2-1)+PHI(I2)))
    GAMXY = ZU/EPS*(-PHI(I1-2)+PHI(I1)+2.*X*(-PHI(I2-2)+PHI(I2)))
    RETURN
END
1680000000
1681000000
1682000000
1683000000
1684000000
1685000000
1686000000
1687000000
1688000000
1689000000
1690000000
1691000000
1692000000
1693000000
1694000000
1695000000
1696000000
1697000000
1698000000
1699000000
1700000000
1701000000
1702000000
1703000000
1704000000
1705000000
1706000000
1707000000
1708000000

```

APPENDIX C - Continued

```

SUBROUTINE FLUTTER
  REAL K,MU
  INTEGER ORD
  COMPLEX CDET,AAC
  COMMON
    1/BLK1/NN,NN1,NN2,NN22,NN23,NN33,MCS,NCS,NMAX
    2/BLK2/CMEGA,V
    3/BLK3/ANG,RHOW,E,MU,C(16),SS,EPS
    4/BLK4/A,RHQA,IHP
    5/BLK5/CMEGAI,DELOM,OMFIN,V1,DELV,VFIN
    1/BLK6/IOPT,IPRINT,ISNDWCH
    8/BLK8/KDERIV
    7/BLK17/NEWR
    1/BLK20/IFIRST,IFAIL,NOCONV,NOS
    6/BLK26/OMSAVE,VSAVE
    1/NEWRBLK/AAC(33,33),AA(33,33),S(5,11),F(5,11),HP(5,11)
    1/BLK31/OCR,OMPLUS,VPLUS
    KOUNT = 0
    NMAX=33
    OMEGA = OMSAVE = OMEGA1
    V = VSAVE = V1
    IF(NEWR.EQ.1) PRINT 61
61  FORMAT(///10X*OMEGA*,17X*V*,24X*DETERMINANT*/
      17X*RADIANS/SEC.*,8X*INCHES/SEC.*,10X*REAL PART*,9X*IMAGINARY PART*
      2/)
7702 CONTINUE

C  COMPLEX DETERMINANT
  CALL AAMAT
  CALL SMAT
  CALL PUTTOG
  ORD=NN33
  IF(IPRINT.EQ.0) RETURN
  DO 101 I=1,ORD
    DO 101 J=1,ORD
      101 AAC(I,J) = (1.E-15,0.)*AAC(I,J)
  CALL DEUPPS(AAC,ORD,CDET,NMAX)
  AO = REAL(CDET)
  RO = AIMAG(CDET)
  IF(NEWR.EQ.1) PRINT 6, CMEGA,V,COET
6  FORMAT(4E20.8)

```

APPENDIX C - Continued

```

IF(IOPT.NE.1) GO TO 2000
C  ITERATE ON OMEGA AND V
V = V+DELV
IF(V.GT.VFIN) GO TO 7701
GO TO 7702
7701 OMEGA = OMEGA+DELOM
IF(OMEGA.GT.OMFIN) STOP
V = V1
GO TO 7702
2000 CONTINUE
OMEGA = OMEGA1+DELOM
CALL PUTTOG
ORD=NN33
DO 102 I=1,ORD
DO 102 J=1,ORD
102 AAC(I,J) = (1.E-15,0.)*AAC(I,J)
CALL DEUPPS(AAC,ORD,CDET,NMAX)
A1 = REAL(CDET)
B1 = AIMAG(CDET)
IF(NEWR.EQ.1) PRINT 6, OMEGA,V,CDET
OMEGA = OMEGA1
V = V1+DELV
CALL PUTTOG
ORD=NN33
DO 103 I=1,ORD
DO 103 J=1,ORD
103 AAC(I,J) = (1.E-15,0.)*AAC(I,J)
CALL DEUPPS(AAC,ORD,CDET,NMAX)
A2 = REAL(CDET)
B2 = AIMAG(CDET)
IF(NEWR.EQ.1) PRINT 6, OMEGA,V,CDET
H = (-A0*(B2-B0)/DELV+B0*(A2-A0)/DELV)/
1((A1-A0)/DELOM*(B2-B0)/DELV-(B1-B0)/DELOM*(A2-A0)/DELV)
K = (-B0*(A1-A0)/DELOM+A0*(B1-B0)/DELOM)/
1((A1-A0)/DELOM*(B2-B0)/DELV-(B1-B0)/DELOM*(A2-A0)/DELV)
OMEGA = OMEGA1+H
V = V1+K
CALL PUTTOG
ORD=NN33
DO 104 I=1,ORD
DO 104 J=1,ORD

```

```

104 AAC(I,J) = (1.E-15,0.)*AAC(I,J)
    CALL DEUPPS(AAC,ORD,CDET,NMAX)
    AO = REAL(CDET)
    BO = AIMAG(CDET)
    IF(NEWR.EQ.1) PRINT 6, OMEGA,V,CDET
    KOUNT = KOUNT+1
    TEST = 1.E-5
    IF(ABS((OMEGA1-OMEGA)/OMEGA).LT.TEST.AND.ABS((V1-V)/V).LT.TEST)
        1 GO TO 300
    IF(KOUNT.LT.7) GO TO 500
    GO TO(501,502),IFIRST
501 CONTINUE
    PRINT 555
555 FORMAT(/* 7 IMPROVEMENTS ON GMINIT AND VINIT WITHOUT CONVERGENCE*)
    STOP
502 CONTINUE
    IF(IPRINT.NE.1) PRINT 666
666 FORMAT(/* 7 IMPROVEMENTS, NO CONVERGENCE, REDUCE SINC*/)
    NCONV = 1
    OMEGA1 = OMSAVE
    V1 = VSAVE
    RETURN
500 CONTINUE
    OMEGA1 = OMEGA
    V1 = V
    GO TO 2000
300 IF(NEWR.EQ.1) PRINT 301
301 FORMAT(/* COMPLEX DETERMINANT CONVERGED TO 0*)
    Q = .5*RHOA*V**2*144.
    IF(IPRINT.EQ.3.OR.NEWR.EQ.1) PRINT 302, Q
302 FORMAT(/* FLUTTER Q =*E16.8* LB./SQ.FT.*)
    OPLUS = OMEGA
    VPLUS = V
    IF(KDERIV.EQ.0) GO TO 5000
    OMEGA1 = OMSAVE
    V1 = VSAVE
5000 RETURN
    END

```

APPENDIX C - Continued

```

SUBROUTINE DEUPPS(A,N,DET,MAX)
C  EVALUATES COMPLEX DETERMINANT IN FLUTTER
C  DEUPPS = DETERMINANT EVALUATION USING PARTIAL PIVOTAL STRATEGY

      COMPLEX A(MAX,N),SWAP,DET,CO,C1
      CO = (0.,0.)
      C1 = (1.,0.)
      DET = C1
      NN = N-1

C  PIVOT SEARCH

      DO 560 I=1,NN
      CAVM = 0.
      II = I
      DO 105 J=II,N
      SWAP = A(J,II)
      AR = REAL(SWAP)
      AI = AIMAG(SWAP)
      CAVA = ABS(AR)+ABS(AI)
      IF(CAVM.GE.CAVA) GO TO 105
      IROW = J
      CAVM = CAVA
105 CONTINUE
      IF(CAVM.EQ.0.) GO TO 720

C  ROW INTERCHANGE

      IF(IROW.EQ.II) GO TO 203
      DET = -DET
      DO 200 L=II,N
      SWAP = A(IROW,L)
      A(IROW,L) = A(II,L)
      A(II,L) = SWAP
200 CONTINUE
      203 SWAP = A(II,II)
      DET = DET*SWAP

C  NORMALIZE PIVOT ROW

      K = II+1

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1832000000
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1840000000
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1850000000
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1860000000
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1866000000
1867000000
1868000000
1869000000
1870000000
1871000000
1872000000
1873000000

APPENDIX C - Continued

```

DO 350 L=K,N
350 A(II,L) = A(II,L)/SWAP
C  ELIMINATION
      DO 550 L1=K,N
      SWAP = A(L1,II)
      DO 500 L=K,N
      A(L1,L) = A(L1,L)-A(II,L)*SWAP
500 CONTINUE
550 CONTINUE
560 CONTINUE
      GO TO 730
720 DET = CO
      GO TO 750
730 DET = DET*A(N,N)
750 RETURN
      END
187400000
187500000
187600000
187700000
187800000
187900000
188000000
188100000
188200000
188300000
188400000
188500000
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189000000
189100000

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1892000000
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1918000000
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1920000000
1921000000
1922000000
1923000000

```

SUBROUTINE SMAT
C  GENERATES DIAGONAL ELEMENTS IN S MATRIX

REAL MU
COMPLEX AAC
DIMENSION SUMS(5),FOFX(5)
EXTERNAL FUNCS
COMMON
1/NEWBLK/AAC(33,33),AA(33,33),S(5,11),F(5,11),HP(5,11)
1/BLK1/AN,NN1,NN2,NN23,NN23,NN33,MCS,NCS,NMAX
3/BLK3/ANG,RHOW,E,MU,C(16),SS,EPS
1/BLK6/IOPT,IPRINT,ISNDWCH
3/BLK13/Y,C1,C2
DO 100 I=1,NN
Y = EPS*FLOAT(I)
CALL CEE
CALL MGAUSS(C1,C2,NCS,SUMS,FUNCS,FOFX,5)
DO 50 K=1,5
S(K,I)=SUMS(K)
IF (I.EQ.NN) S(K,I)=.5*S(K,I)
50 CONTINUE
100 CONTINUE
DO 200 K=1,5
S(K,NN1)=0.
IF(IPRINT.NE.0) RETURN
PRINT 1, ((S(J,I),J=1,5),I=1,NN1)
1 FORMAT(/////* DIAGONAL ELEMENTS OF S MATRICES*/ PRINTED HERE AS CC
1LUMNS SO-S4 (PROGRAM NOTATION IS S(1,I)-S(5,I)), ROWS ARE STATIONS
1 Y=1,NN1*/(5E16.8))
RETURN
END

```

APPENDIX C - Continued

```

SUBROUTINE FMAT
C   GENERATES DIAGONAL ELEMENTS IN F MATRIX

1  FORMAT(1H1// * DIAGONAL ELEMENTS OF F MATRICES*/* PRINTED HERE AS C
1  COLUMNS FO-F4 (PROGRAM NOTATION IS F(1,I)-F(5,I))*/* ROWS ARE STATI
1  IONS Y=1,NN1*//(5E16.8))
REAL MU
COMPLEX AAC
COMMON
1  NEWBLK/AAC(33,33),AA(33,33),S(5,11),F(5,11),HP(5,11)
1  BLK1/NN,NN1,NN2,NN23,NN23,MCS,NCS,NMAX
1  BLK3/ANG,RHOW,E,MU,C(16),SS,EPS
1  BLK6/IOPT,IPRINT,ISNDWCH
1  BLK13/Y,C1,C2
DO 100 I=1,NN
Y=EPS*FLOAT(I)
CALL CEE
DO 50 K=1,5
XK=K
F(K,I)=1./XK*(C2**K-C1**K)
IF(I.EQ.NN) F(K,I)=.5*F(K,I)
50 CONTINUE
100 CONTINUE
DO 200 K=1,5
F(K,NN1)=0.
IF(IPRINT.NE.0) RETURN
PRINT 1, ((F(J,I),J=1,5),I=1,NN1)
RETURN
END
192400000
192500000
192600000
192700000
192800000
192900000
193000000
193100000
193200000
193300000
193400000
193500000
193600000
193700000
193800000
193900000
194000000
194100000
194200000
194300000
194400000
194500000
194600000
194700000
194800000
194900000
195000000
195100000
195200000
195300000

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APPENDIX C - Continued

```

SUBROUTINE HMAT
C  GENERATES DIAGONAL ELEMENTS IN HP MATRIX

1  FORMAT(/////* DIAGONAL ELEMENTS OF HP MATRICES*/ * PRINTED HERE AS
1  COLUMNS HP1-HP4 (PROGRAM NOTATION IS HP(2,I)-HP(5,I)). ROWS ARE ST
1  IATIONS Y=1,NN1*/ * HPO=0 AND IS NOT CALCULATED*/(4E16.8))
REAL MU
COMPLEX AAC
COMMON
1/NEWBLK/AAC(33,33),AA(33,33),S(5,11),F(5,11),HP(5,11)
1/BLK1/NN,NN1,NN2,NN23,NN23,MCS,NCS,NMAX
1/BLK3/ANG,RHOW,E,MU,C(16),SS,EPS
1/BLK4/A,RHOA,IHP
1/BLK6/IOPT,IPRINT,ISNDWCH
1/BLK13/Y,C1,C2
1/BLK18/DR
DO 100 I=1,NN
Y=EPS*FLOAT(I)
CALL CEE
IF(IHP.EQ.0) GO TO 150
DO 120 N=2,5
FN=FLOAT(N-1)
NEW=N-1
NEW2=NEW+2
HP(N,I)=-8.*DR/(C2-C1)*((FN*(C2**NEW2-C1**NEW2)/(2.*(FN+2.))*((FN+1.))
1)+C1*C2*(C1**NEW-C2**NEW)/(2.*(FN+1.)))
IF(I.EQ.NN) HP(N,I)=.5*HP(N,I)
120 CONTINUE
GO TO 100
150 CONTINUE
HP(2,I)=HP(3,I)=HP(4,I)=HP(5,I)=0.
100 CONTINUE
DO 200 N=2,5
200 HP(N,NN1)=0.
IF(IPRINT.NE.0) RETURN
PRINT 1, ((HP(J,I),J=2,5),I=1,NN1)
RETURN
END

```

```

SUBROUTINE PUTTOG
1  FORMAT(1H1// * AAC MATRIX*,5X13,* X*13,* COMPLEX AA MATRIX AUGMENT
   LED WITH INERTIA TERMS (S MATRICES) AND PISTON THEORY (F AND HP AR
   2RAYS)*//)
2  FORMAT(1X10E11.4)
4  FORMAT(1H1//////)
21 FORMAT(* PISTON THEORY USES OMEGA =*E16.8,* AND V =*E16.8////)

REAL MU
COMPLEX AAC
COMMON
1/NEWBLK/AAC(33,33),AA(33,33),S(5,11),F(5,11),HP(5,11)
1/BLK1/AN,NN1,NN2,NN22,NN23,NN33,MCS,NCS,NMAX
2/BLK2/OMEGA,V
1/BLK3/ANG,RHOW,E,MU,C(16),SS,EPS
1/BLK4/A,RHOA,IHP
1/BLK6/IQPT,IPRINT,ISNDWCH
OM2=CMEGA**2
EPS4=EPS**4
GAM=1.4
GAM14=(GAM+1.)/4.
DO 100 I=1,NN33
DO 100 J=1,NN33
100 AAC(I,J)=AA(I,J)
INNI=I+NN1
INN22=I+NN22
AREAL=AA(I,I)+EPS4*(-CM2*S(1,I))
AMAG=EPS4*(2.*RHOA*A*OMEGA*F(1,I))
AAC(I,I)=CMPLX(AREAL,AMAG)
AREAL=AA(I,INNI)+EPS4*(-CM2*S(2,I)+2.*RHOA*A*V*F(1,I))
AMAG=EPS4*(2.*RHOA*A*OMEGA*F(2,I)+2.*RHOA*V*GAM14*OMEGA*HP(2,I))
AAC(I,INNI)=CMPLX(AREAL,AMAG)
AREAL=AA(I,INN22)+EPS4*(-CM2*S(3,I)+4.*RHOA*A*V*F(2,I)+4.*RHCA*V
1*V*GAM14*HP(2,I))
AMAG=EPS4*(2.*RHOA*A*OMEGA*F(3,I)+2.*RHOA*V*GAM14*OMEGA*HP(3,I))
AAC(I,INN22)=CMPLX(AREAL,AMAG)
AREAL=AA(INNI,I)+EPS4*(-CM2*S(2,I))
AMAG=EPS4*(2.*RHOA*A*OMEGA*F(2,I)+V*GAM14/A*HP(2,I))
AAC(INNI,I)=CMPLX(AREAL,AMAG)
AREAL=AA(INNI,INNI)+EPS4*(-OM2*S(3,I)+2.*RHOA*A*V*F(2,I)

```

APPENDIX C - Continued

```

1+2.*RHOA*V*VGAM14*HP(2,I)
AMAG=EPS4*(2.*RHOA*A*OMEGA*F(3,I)+2.*RHOA*V*VGAM14*OMEGA*HP(3,I))
AAC(INN1,INN1)=CMPLX(AREAL,AMAG)
AREAL=AA(INN1,INN2)+EPS4*(-OM2*S(4,I)+4.*RHOA*A*V*F(3,I)
1+4.*RHCA*V*VGAM14*HP(3,I)
AMAG=EPS4*(2.*RHOA*A*OMEGA*F(4,I)+2.*RHOA*V*VGAM14*OMEGA*HP(4,I))
AAC(INN1,INN2)=CMPLX(AREAL,AMAG)
AREAL=AA(INN2,I)+EPS4*(-CM2*S(3,I))
AMAG=EPS4*(2.*RHOA*A*OMEGA*F(3,I)+V*VGAM14/A*HP(3,I))
AAC(INN2,I)=CMPLX(AREAL,AMAG)
AREAL=AA(INN2,INN1)+EPS4*(-OM2*S(4,I)+2.*RHOA*A*V*F(3,I)
1+2.*RHOA*V*VGAM14*HP(3,I)
AMAG=EPS4*(2.*RHOA*A*OMEGA*F(4,I)+2.*RHOA*V*VGAM14*OMEGA*HP(4,I))
AAC(INN2,INN1)=CMPLX(AREAL,AMAG)
AREAL=AA(INN2,INN2)+EPS4*(-OM2*S(5,I)+4.*RHOA*A*V*F(4,I)
1+4.*RHOA*V*VGAM14*HP(4,I)
AMAG=EPS4*(2.*RHOA*A*OMEGA*F(5,I)+2.*RHOA*V*VGAM14*OMEGA*HP(5,I))
AAC(INN2,INN2)=CMPLX(AREAL,AMAG)
200 CONTINUE
IF(IPRINT.NE.0) RETURN
PRINT 1, NN33,NN33
PRINT 21, OMEGA,V
PRINT 2, ((AAC(I,J),J=1,NN1),I=1,NN33)
PRINT 4
PRINT 2, ((AAC(I,J),J=NN2,NN22),I=1,NN33)
PRINT 4
PRINT 2, ((AAC(I,J),J=NN23,NN33),I=1,NN33)
RETURN
END
2035000000
2036000000
2037000000
2038000000
2039000000
2040000000
2041000000
2042000000
2043000000
2044000000
2045000000
2046000000
2047000000
2048000000
2049000000
2050000000
2051000000
2052000000
2053000000
2054000000
2055000000
2056000000
2057000000
2058000000
2059000000
2060000000
2061000000
2062000000
2063000000

```

APPENDIX C - Continued

206400000
206500000
206600000
206700000
206800000
206900000
207000000
207100000
207200000
207300000
207400000

SUBROUTINE CEE

C DEFINES LEADING EDGE (C1) AND TRAILING EDGE (C2)

 COMMON
 3/BLK36/C1A,C1B,C2A
 3/BLK13/Y,C1,C2
 C1 = C1A+C1B*Y
 C2 = C2A
 RETURN
 END

APPENDIX C - Continued

```

SUBROUTINE AAMAT
1  FORMAT(1H1// * AA MATRIX*,5X13,* X*13,5X* A MATRIX IN EQ. 29, PG.
1 13 NACA TN-3640*//)
2  FORMAT(5E16.8)
5  FORMAT(1H1/////)
10 FORMAT(1H1// * THE FOLLOWING MATRICES ARE PRINTED FROM SUBROUTINES C
1A1MAT AND AAMAT*/////)

    DIMENSION A(11,11),AT(11,11)
    COMPLEX AAC
    COMMON
1/BLK1/NN,NN1,NN2,NN23,NN23,NN33,MCS,NCS,NMAX
1/BLK6/IOPT,IPRINT,ISNOWCH
1/NEWBLK/AAC(33,33),AA(33,33),S(5,11),F(5,11),HP(5,11)
    ITRANS = 0
    MOVE = 1

C   NA = 1   AA(1,1) TO AA(NN1,NN1) = A11
C   NA = 2   AA(1,NN2) TO AA(NN1,NN22) = A12
C   NA = 3   AA(NN2,1) TO AA(NN22,NN1) = A21
C   NA = 3   AA(1,NN23) TO AA(NN1,NN23) = A13
C   NA = 4   AA(NN23,1) TO AA(NN33,NN1) = A31
C   NA = 4   AA(NN2,NN2) TO AA(NN22,NN22) = A22
C   NA = 5   AA(NN2,NN23) TO AA(NN22,NN33) = A23
C   NA = 5   AA(NN23,NN2) TO AA(NN34,NN23) = A32
C   NA = 6   AA(NN23,NN23) TO AA(NN33,NN33) = A33

    IF(IPRINT.EQ.0) PRINT 10
    DO 100 NA=1,6
    CALL AAMAT(NA,A)
    IF(NA.EQ.4.OR.NA.EQ.5) GO TO 104
    IF(NA.EQ.6) GO TO 106
    DO 110 I=1,NN1
    DO 110 J=1,NN1
    JA = J
    IF(NA.EQ.2) JA = J+NN1
    IF(NA.EQ.3) JA = J+NN22
    AAC(I,JA) = AA(I,JA) = A(I,J)
110 CONTINUE
    GO TO (100,200,200), NA
104 DO 140 I=1,NN1

```

APPENDIX C - Continued

```

211700000
211800000
211900000
212000000
212100000
212200000
212300000
212400000
212500000
212600000
212700000
212800000
212900000
213000000
213100000
213200000
213300000
213400000
213500000
213600000
213700000
213800000
213900000
214000000
214100000
214200000
214300000
214400000
214500000
214600000
214700000
214800000

DO 140 J=1,NN1
JA = J
IF(NA.EQ.4) JA=J+NN1
IF(NA.EQ.5) JA=J+NN2
IA = I+NN1
AAC(IA,JA) = AA(IA,JA) = A(I,J)
140 CONTINUE
IF(NA.EQ.5) GO TO 200
GO TO 100
106 DO 160 I=1,NN1
DO 160 J=1,NN1
JA = J
IF(NA.EQ.5) JA=J+NN1
IF(NA.EQ.5) JA = J+NN2
IA = I+NN2
AAC(IA,JA) = AA(IA,JA) = A(I,J)
160 CONTINUE
GO TO 100
200 CALL MATRIX(ITRANS,NN1,NN1,0,A,MCS,AT,MCS)
CALL MATRIX(MOVE,NN1,NN1,C,AT,MCS,A,MCS)
IF(NA.EQ.2) GO TO 104
GO TO 106
100 CONTINUE
IF(IPRINT.NE.0) RETURN
PRINT 1, NN3,NN33
PRINT 2, ((AA(I,J),J=1,NN1),I=1,NN33)
PRINT 5
PRINT 2, ((AA(I,J),J=NN2,NN22),I=1,NN33)
PRINT 5
PRINT 2, ((AA(I,J),J=NN23,NN33),I=1,NN33)
RETURN
END

```

APPENDIX C - Continued

```

SUBROUTINE AMAT(NA,DUM1)
1  FORMAT(//* AMAT      NA =*13/)
2  FORMAT(5E16.8)

REAL MU
DIMENSION DUM1(11,11),DUM2(11,11),DUM3(11,11)
COMMON
1/RLK1/NN,NN1,NN2,NN22,NN23,NN33,MCS,NCS,NMAX
2/RLK2/CMEGA,V
3/RLK3/ANG,RHOW,E,MU,C(16),SS,EPS
1/RLK6/IOP,T,I,PRINT,ISNDWCH
MULT = 20
ISCLR = 24
ISUB = 22
IADD = 21
ITRANS = 0
SCLR = 2.*MU*EPS**2
SCLR2 = 2.*(1.-MU)*EPS**2
SCLR4 = 4.*(1.-MU)*EPS**2
SCLR8 = 8.*(1.-MU)*EPS**2
SCLR4E = 4.*EPS**4
GO TO (11,12,13,22,23,33),NA

C    A11 = D1*A1*D1T

11 CALL CALMAT(1,1,DUM1,NN1,NN1)
   CALL CALMAT(3,1,DUM2,NN1,NN1)
   CALL MATRIX(MULT,NN1,NN1,DUM1,MCS,DUM2,MCS,DUM1,MCS)
   CALL CALMAT(2,1,DUM2,NN1,NN1)
   CALL MATRIX(MULT,NN1,NN1,DUM1,MCS,DUM2,MCS,DUM1,MCS)
   IF(I,PRINT,NE.0) RETURN
   PRINT 1, NA
   PRINT 2, ((DUM1(I,J),J=1,NN1),I=1,NN1)
   RETURN

C    A12 = D1*A2*D1T

12 CALL CALMAT(1,1,DUM1,NN1,NN1)
   CALL CALMAT(3,2,DUM2,NN1,NN1)
   CALL MATRIX(MULT,NN1,NN1,DUM1,MCS,DUM2,MCS,DUM1,MCS)
   CALL CALMAT(2,1,DUM2,NN1,NN1)

```

214900000
215000000
215100000
215200000
215300000
215400000
215500000
215600000
215700000
215800000
215900000
216000000
216100000
216200000
216300000
216400000
216500000
216600000
216700000
216800000
216900000
217000000
217100000
217200000
217300000
217400000
217500000
217600000
217700000
217800000
217900000
218000000
218100000
218200000
218300000
218400000
218500000
218600000
218700000
218800000
218900000
219000000

```

CALL MATRIX(MULT,NN1,NN1,NN1,DUM1,MCS,DUM2,MCS,DUM1,MCS)
IF(IPRINT.NE.0) RETURN
PRINT 1, NA
PRINT 2, ((DUM1(I,J),J=1,NN1),I=1,NN1)
RETURN

C   A13 = D1*(A3*D1T+SCLR*A1BAR)

13  CALL CALMAT(3,3,DUM1,NN1,NN1)
    CALL CALMAT(2,1,DUM2,NN1,NN1)
    CALL MATRIX(MULT,NN1,NN1,NN1,DUM1,MCS,DUM2,MCS,DUM1,MCS)
    CALL CALMAT(4,1,DUM2,NN1,NN1)
    CALL MATRIX(ISCLR,NN1,NN1,0,SCLR,0,DUM2,MCS,DUM2,MCS)
    CALL MATRIX(IADD,NN1,NN1,0,DUM1,MCS,DUM2,MCS,DUM2,MCS)
    CALL CALMAT(1,1,DUM1,NN1,NN1)
    CALL MATRIX(MULT,NN1,NN1,NN1,DUM1,MCS,DUM2,MCS,DUM1,MCS)
    IF(IPRINT.NE.0) RETURN
    PRINT 1, NA
    PRINT 2, ((DUM1(I,J),J=1,NN1),I=1,NN1)
    RETURN

C   A22 = D1*A3*D1T+SCLR*D4*A1STR*D4T

22  CALL CALMAT(1,1,DUM1,NN1,NN1)
    CALL CALMAT(3,3,DUM2,NN1,NN1)
    CALL MATRIX(MULT,NN1,NN1,NN1,DUM1,MCS,DUM2,MCS,DUM1,MCS)
    CALL CALMAT(2,1,DUM2,NN1,NN1)
    CALL MATRIX(MULT,NN1,NN1,NN1,DUM1,MCS,DUM2,MCS,DUM1,MCS)
    CALL CALMAT(1,4,DUM2,NN1,NN1)
    CALL CALMAT(5,1,DUM3,NN,NN)
    CALL MATRIX(MULT,NN1,NN,NN,DUM2,MCS,DUM3,MCS,DUM2,MCS)
    CALL CALMAT(2,4,DUM3,NN1,NN1)
    CALL MATRIX(MULT,NN1,NN,NN1,DUM2,MCS,DUM3,MCS,DUM2,MCS)
    CALL MATRIX(ISCLR,NN1,NN1,0,SCLR2,0,DUM2,MCS,DUM2,MCS)
    CALL MATRIX(IADO,NN1,NN1,0,DUM1,MCS,DUM2,MCS,DUM1,MCS)
    IF(IPRINT.NE.0) RETURN
    PRINT 1, NA
    PRINT 2, ((DUM1(I,J),J=1,NN1),I=1,NN1)
    RETURN

C   A23 = D1*A4*D1T+SCLR4*D4*A2STR*D4T+SCLR*D1*A2BAR

23  CALL CALMAT(1,1,DUM1,NN1,NN1)

```


APPENDIX C - Continued

```

CALL CALMAT(3,4,DUM2,NN1,NN1)
CALL MATRIX(MULT,NN1,NN1,NN1,DUM1,MCS,DUM2,MCS,DUM1,MCS)
CALL CALMAT(2,1,DUM2,NN1,NN1)
CALL MATRIX(MULT,NN1,NN1,NN1,DUM1,MCS,DUM2,MCS,DUM1,MCS)
CALL CALMAT(1,4,DUM2,NN1,NN)
CALL CALMAT(5,2,DUM3,NN,NN)
CALL MATRIX(MULT,NN1,NN,NN,DUM2,MCS,DUM3,MCS,DUM2,MCS)
CALL CALMAT(2,4,DUM3,NN1,NN)
CALL MATRIX(MULT,NN1,NN,NN1,DUM2,MCS,DUM3,MCS,DUM2,MCS)
CALL MATRIX(ISCLR,NN1,NN1,0,SCLR4,0,DUM2,MCS,DUM2,MCS)
CALL MATRIX(IADD,NN1,NN1,0,DUM1,MCS,DUM2,MCS,DUM1,MCS)
CALL CALMAT(1,1,DUM2,NN1,NN1)
CALL CALMAT(4,2,DUM3,NN1,NN1)
CALL MATRIX(MULT,NN1,NN1,NN1,DUM2,MCS,DUM3,MCS,DUM2,MCS)
CALL MATRIX(ISCLR,NN1,NN1,0,SCLR,0,DUM2,MCS,DUM2,MCS)
CALL MATRIX(IADD,NN1,NN1,0,DUM1,MCS,DUM2,MCS,DUM1,MCS)
IF(IPRINT.NE.0) RETURN
PRINT 1, NA
PRINT 2, ((DUM1(I,J),J=1,NN1),I=1,NN1)
RETURN
C      A33 = D1*A5*D1T+SCLR8*D4*A3STR*D4T+SCLR*(D1*A3BAR+(D1*A3BAR)T)
C      +SCLR4E*A1HAT
33 CALL CALMAT(1,1,DUM1,NN1,NN1)
CALL CALMAT(3,5,DUM2,NN1,NN1)
CALL MATRIX(MULT,NN1,NN1,NN1,DUM1,MCS,DUM2,MCS,DUM1,MCS)
CALL CALMAT(2,1,DUM2,NN1,NN1)
CALL MATRIX(MULT,NN1,NN1,NN1,DUM1,MCS,DUM2,MCS,DUM1,MCS)
CALL CALMAT(1,4,DUM2,NN1,NN)
CALL CALMAT(5,3,DUM3,NN,NN)
CALL MATRIX(MULT,NN1,NN,NN,DUM2,MCS,DUM3,MCS,DUM2,MCS)
CALL CALMAT(2,4,DUM3,NN1,NN)
CALL MATRIX(MULT,NN1,NN,NN1,DUM2,MCS,DUM3,MCS,DUM2,MCS)
CALL MATRIX(ISCLR,NN1,NN1,0,SCLR8,0,DUM2,MCS,DUM2,MCS)
CALL MATRIX(IADD,NN1,NN1,0,DUM1,MCS,DUM2,MCS,DUM1,MCS)
CALL CALMAT(1,1,DUM2,NN1,NN1)
CALL CALMAT(4,3,DUM3,NN1,NN1)
CALL MATRIX(MULT,NN1,NN1,NN1,DUM2,MCS,DUM3,MCS,DUM2,MCS)
CALL MATRIX(ITRANS,NN1,NN1,0,DUM2,MCS,DUM3,MCS)
CALL MATRIX(IADD,NN1,NN1,0,DUM2,MCS,DUM3,MCS,DUM2,MCS)
CALL MATRIX(ISCLR,NN1,NN1,0,SCLR,0,DUM2,MCS,DUM2,MCS)
CALL MATRIX(IADD,NN1,NN1,0,DUM1,MCS,DUM2,MCS,DUM1,MCS)

```

APPENDIX C - Continued

```

227700000
227800000
227900000
228000000
228100000
228200000
228300000
228400000

```

```

CALL CALMAT(6,1,DUM2,NN1,NN1)
CALL MATRIX(ISCLR,NN1,NN1,0,SCLR4E,0,DUM2,MCS,DUM2,MCS)
CALL MATRIX(IADD,NN1,NN1,0,DUM1,MCS,DUM2,MCS,DUM1,MCS)
IF(IPRINT.NE.0) RETURN
PRINT 1, NA
PRINT 2, ((DUM1(I,J),J=1,NN1),I=1,NN1)
RETURN
END

```

APPENDIX C - Continued

```

SUBROUTINE MINGAGE(G3)
C  EVALUATES MINIMUM GAGE CONSTRAINT

EXTERNAL FTHICK
REAL MU
DIMENSION TX(11),FOFX(1)
DIMENSION ANS(1)
COMMON
1/BLK1/NN,NN1,NN2,NN22,NN23,NN33,MCS,NCS,NMAX
3/BLK3/ANG,RHOW,E,MU,C(16),SS,EPS
1/BLK13/Y,C1,C2
1/BLK20/IFIRST,IFAIL,NOCONV,NCS
1/BLKNPS/AREA
1/NPSBLK/ITHICK
IF(ITHICK.EQ.1) INDEX=NN1
IF(ITHICK.EQ.2) INDEX=NN
DO 500 I=1,INDEX
Y = EPS*FLOAT(I-1)
CALL CEE
CALL MGAUSS(C1,C2,NCS,ANS,FTHICK, FOFX,1)
IF(IFAIL.EQ.1) RETURN
500 TX(I) = ANS(I)
IF(ITHICK.EQ.1) SUM=1./2.*TX(1)+1./2.*TX(NN1)
IF(ITHICK.EQ.2) SUM=1./2.*TX(1)
DO 200 I=2,NN
200 SUM = SUM+TX(I)
SUM = SUM*EPS/AREA
G3 = 1.-1./SUM
RETURN
END
2285000000
2286000000
2287000000
2288000000
2289000000
2290000000
2291000000
2292000000
2293000000
2294000000
2295000000
2296000000
2297000000
2298000000
2299000000
2300000000
2301000000
2302000000
2303000000
2304000000
2305000000
2306000000
2307000000
2308000000
2309000000
2310000000
2311000000
2312000000
2313000000
2314000000
2315000000

```

APPENDIX C - Continued

```

SUBROUTINE FTHICK(X,FOFX)
  DIMENSION FCFX(1)
  COMMON
  1/BLK6/IOPT,IPRINT,ISNCWCH
  7/BLK7/TBAR
  7/BLK17/NEWR
  1/BLK20/IFIRST,IFAIL,NOCONV,NQS
  3/BLK25/TBARMIN
  CALL THKNES(X)
  IF(TBAR.GE.TBARMIN) GO TO 200
  IF(IFIRST.EQ.1) PRINT 10
  10 FORMAT(/* INITIAL DESIGN UNACCEPTABLE BECAUSE THICKNESS IS LESS TH
    1AN MINIMUM GAGE*)
    IF(IFIRST.EQ.1) STOP
    IFAIL=1
    IF(IPRINT.EQ.3.OR.NEWR.EQ.1) PRINT 11
    11 FORMAT(/* THICKNESS LESS THAN MINIMUM GAGE - REDUCE S OR DC*/)
    RETURN
  200 CONTINUE
  FOFX(1) = 1./(1.-TBARMIN/TBAR)
  100 CONTINUE
  RETURN
  END
231600000
231700000
231800000
231900000
232000000
232100000
232200000
232300000
232400000
232500000
232600000
232700000
232800000
232900000
233000000
233100000
233200000
233300000
233400000
233500000
233600000
233700000
233800000

```

APPENDIX C - Continued

```

SUBROUTINE THKNSS(X)
REAL MU
COMMON
3/BLK3/ANG,RHOW,E,MU,C(16),SS,EPS
1/BLK13/Y,C1,C2
7/BLK7/TBAR
1/NPSBLK/ITHICK
A = 1.-Y/SS
B = 1.-2.*X/(C1+C2)
IF(ITHICK.EQ.2) GO TO 100
TBAR = C( 1) + C( 2)*A + C( 3)*A**2 + C( 4)*A**3
1      +(C( 5) + C( 6)*A + C( 7)*A**2 + C( 8)*A**3)*B
1      +(C( 9) + C(10)*A + C(11)*A**2 + C(12)*A**3)*B**2
1      +(C(13) + C(14)*A + C(15)*A**2 + C(16)*A**3)*B**3
RETURN
100 CONTINUE
TBAR = C( 1) + C( 2)*A + C( 3)*A**2 + C( 4)*A**3
1      +(C( 5) + C( 6)*A + C( 7)*A**2 + C( 8)*A**3)*B
1      +(C( 9) + C(10)*A + C(11)*A**2 + C(12)*A**3)*(1-B*B)
1      +(C(13) + C(14)*A + C(15)*A**2 + C(16)*A**3)*(1-B*B)*B
RETURN
END
233900000
234000000
234100000
234200000
234300000
234400000
234500000
234600000
234700000
234800000
234900000
235000000
235100000
235200000
235300000
235400000
235500000
235600000
235700000
235800000
235900000
236000000

```

```

SUBROUTINE CALMAT(NAME,NSUB,DUM,IRO ,ICO )
1  FORMAT(//,* NAME =*I3,*   NSUB =*I3/)
2  FORMAT(5E16.8)
3  FORMAT(4E16.8)
4  FORMAT(6E16.8)

  DIMENSION FOFX(1)
  DIMENSION SUMA(1),SUMS(1)
  DIMENSION DUM(11,11),D(11,11),DT(11,11),A(11,11)
  REAL MU
  EQUIVALENCE (D,DT,A)
  EXTERNAL FUNCA
  COMMON
  1/BLK1/M,N,N1,NN2,NN22,NN23,NN33,MCS,NCS,NMAX
  3/BLK3/ANG,RHOW,E,MU,C(16),SS,EPS
  1/BLK6/IOPT,IPRINT,ISNDWCH
  9/BLK9/NSUBX
  1/BLK13/Y,C1,C2

C   NAME = 1  D
C   NAME = 2  DT
C   NAME = 3  A
C   NAME = 4  ABAR
C   NAME = 5  ASTAR
C   NAME = 6  AHAT

  NSUBX=NSUB
  IF(NAME.GT.2) GO TO 300
  IF(NSUB.EQ.4) GO TO 140

C   CALCULATE D1 MATRIX

110 DO 111 I=1,IRO
    DO 111 J=1,ICO
      D(I,J) = 0.
      IF(J.EQ.1) D(I,J) = 1.
      IF(J.EQ.I+1) D(I,J) = -2.
      IF(J.EQ.I+2) D(I,J) = 1.
111 CONTINUE
      D(1,1) = 2.
      IF(NAME.EQ.2) GO TO 200

```

236100000
236200000
236300000
236400000
236500000
236600000
236700000
236800000
236900000
237000000
237100000
237200000
237300000
237400000
237500000
237600000
237700000
237800000
237900000
238000000
238100000
238200000
238300000
238400000
238500000
238600000
238700000
238800000
238900000
239000000
239100000
239200000
239300000
239400000
239500000
239600000
239700000
239800000
239900000
240000000
240100000
240200000

APPENDIX C - Continued

```

IF(I.PRINT.NE.0) GO TO 500
PRINT 1, NAME, NSUB
PRINT 2, ((D(I,J), J=1, ICO), I=1, IRO)
GO TO 500

C      CALCULATE D4 MATRIX

140 DO 141 I=1, IRO
DO 141 J=1, ICO
D(I,J) = 0.
IF(J.EQ.1) D(I,J) = 1.
IF(J.EQ.I+1) D(I,J) = -1.
141 CONTINUE
IF(NAME.EQ.2) GO TO 200
IF(I.PRINT.NE.0) GO TO 500
PRINT 1, NAME, NSUB
PRINT 3, ((D(I,J), J=1, ICO), I=1, IRO)
GO TO 500
200 CONTINUE

C      CALCULATE DT MATRIX

ITRANS = 0
CALL MATRIX(ITRANS, IRO, ICO, 0, D, MCS, DUM, MCS)
IF(I.PRINT.NE.0) RETURN
PRINT 1, NAME, NSUB
DO 31 I=1, ICO
31 PRINT 4, (DUM(I,J), J=1, IRC)
RETURN
300 CONTINUE

C      CALCULATE A MATRIX

DO 301 I=1, IRO
DO 301 J=1, ICO
A(I,J) = 0.
IF(J.NE.1) GO TO 301
N = J-1
Y = EPS*FLOAT(N)
IF(NAME.EQ.5) Y = EPS*(FLOAT(N)+1./2.)
CALL CEE
CALL MGAUSS(C1, C2, NCS, SUMA, FUNCA, FOFX, 1)
A(I,J) = SUMA(1)

```

240300000
240400000
240500000
240600000
240700000
240800000
240900000
241000000
241100000
241200000
241300000
241400000
241500000
241600000
241700000
241800000
241900000
242000000
242100000
242200000
242300000
242400000
242500000
242600000
242700000
242800000
242900000
243000000
243100000
243200000
243300000
243400000
243500000
243600000
243700000
243800000
243900000
244000000
244100000
244200000
244300000
244400000
244500000

APPENDIX C - Continued

```

IF(NAME.EQ.5) GO TO 301
IF(I.EQ.1.OR.I.EQ.NN1) A(I,J) = 1./2.*A(I,J)
301 CONTINUE
IF(NAME.EQ.4) GO TO 310
IF(NAME.EQ.6) GO TO 320
GO TO 400
C SHIFT TO FORM ABAR MATRIX
310 DO 312 I=2,IRO
J=I-1
A(I,J)=A(I,I)
312 A(I,I)=0.
A(1,1)=0.
GO TO 400
C SHIFT TO FORM AHAT MATRIX
320 IRO1=IRO-1
DO 322 I=1,IRO1
322 A(I,I)=A(I+1,I+1)
A(IRO,IRO)=0.
400 CONTINUE
IF(1.PRINT.NE.0) GO TO 500
PRINT 1, NAME,NSUB
DO 41 I=1,IRO
41 PRINT 4, (A(I,J),J=1,ICO)
500 MOVE = 1
CALL MATRIX(MOVE,IRO ,ICO ,0,0,MCS,DUM,MCS)
RETURN
END
244600000
244700000
244800000
244900000
245000000
245100000
245200000
245300000
245400000
245500000
245600000
245700000
245800000
245900000
246000000
246100000
246200000
246300000
246400000
246500000
246600000
246700000
246800000
246900000
247000000
247100000
247200000
247300000
247400000
247500000
247600000
247700000

```


APPENDIX C - Continued

```

SUBROUTINE FUNCA(X,FOFX)
C  EVALUATES BENDING STIFFNESS D
      DIMENSION FOFX(1)
      REAL MU
      COMMON
      3/BLK3/ANG,RHOW,E,MU,C(16),SS,EPS
      1/BLK6/IOPT,IPRINT,ISNDWCH
      7/BLK7/TBAR
      9/BLK9/NSUB
      CALL THKNES(X)
      IF(ISNDWCH.EQ.1) GO TO 100
      D = E*TBAR**3/(12.*(1.-MU**2))
      GO TO 200
100  CALL DEPTH(H,X)
      D = E*H**2/2.*TBAR/(1.-MU**2)+2.*(E*TBAR**3/(12.*(1.-MU**2)))
200  CONTINUE
      FOFX(1) = D*X**(NSUB-1)
      RETURN
      END
2478000000
2479000000
2480000000
2481000000
2482000000
2483000000
2484000000
2485000000
2486000000
2487000000
2488000000
2489000000
2490000000
2491000000
2492000000
2493000000
2494000000
2495000000
2496000000
2497000000
2498000000

```

APPENDIX C - Continued

```

SUBROUTINE FUNCS(X,FOFX)
C  EVALUATES MASS PER UNIT AREA M

REAL M
REAL MU
DIMENSION FOFX(1)
COMMON
3/BLK3/ANG,RHOW,E,MU,C(16),SS,EPS
1/BLK6/IOPT,IPRINT,ISNDWCH
7/BLK7/TBAR
1/BLK50/RHOF
CALL THKNES(X)
IF(ISNDWCH.EQ.1) GO TO 100
M = RHCW*TBAR
GO TO 200
100 CALL DEPTH(H,X)
M = 2.*RHOW*TBAR+RHDF*H
200 CONTINUE
DO 300 I=1,5
300 FOFX(I)=M*X**(I-1)
RETURN
END
249900000
250000000
250100000
250200000
250300000
250400000
250500000
250600000
250700000
250800000
250900000
251000000
251100000
251200000
251300000
251400000
251500000
251600000
251700000
251800000
251900000
252000000
252100000

```

APPENDIX C - Continued

```

SUBROUTINE DEPTH(H,X)
C  AIRFOIL MUST BE SYMMETRIC - THAT IS, NO CAMBER.
COMMON
1/BLK13/Y,C1,C2
1/BLK6/IOPT,IPRINT,ISNDWCH
7/BLK7/TBAR
8/BLK18/D
IF (ISNDWCH.EQ.0) GO TO 100
H=D*(C2-C1)-4.*D/(C2-C1)*(X-(C2+C1)/2.)*2
RETURN
100 CALL THKNES(X)
H = TBAR
RETURN
END
252200000
252300000
252400000
252500000
252600000
252700000
252800000
252900000
253000000
253100000
253200000
253300000
253400000
253500000
253600000
253700000

```

```

SUBROUTINE TABLE
1 FORMAT(1H1///* THICKNESS TABLE*// TBAR*//)
1* Y X
2 FORMAT(F6.0,2E20.8)

REAL MU
COMMON
1/BLK1/NN,NN1,NN2,NN23,NN33,MCS,NCS,NMAX
3/BLK3/ANG,RHOW,E,MU,C(16),SS,EPS
7/BLK7/TBAR
3/BLK13/Y,C1,C2
PRINT 1
YINC = SS/(2.*FLOAT(NN))
Y = 0.
200 CONTINUE
CALL CEE
XINC = (C2-C1)/(20.*FLOAT(NCS))
X = C1
100 CONTINUE
CALL THKNES(X)
PRINT 2, Y,X,TBAR
X = X+XINC
IF(X.GT.(C2+.0001)) GO TO 101
GO TO 100
101 CONTINUE
Y = Y+YINC
IF(Y.GT.SS) RETURN
GO TO 200
END

```

2538000000
2539000000
2540000000
2541000000
2542000000
2543000000
2544000000
2545000000
2546000000
2547000000
2548000000
2549000000
2550000000
2551000000
2552000000
2553000000
2554000000
2555000000
2556000000
2557000000
2558000000
2559000000
2560000000
2561000000
2562000000
2563000000
2564000000
2565000000
2566000000
2567000000

APPENDIX C - Continued

```

SUBROUTINE ITR2 (X,A,B,DELTX,FOFX,E1,E2,MAXI,ICODE)
***** DOCUMENT DATE 08-01-68 SUBROUTINE REVISED 08-01-68 *****
X=A
KX=0
LX=0
IF (DELTX)111,111,112
112 IF (B-A) 113,113,114
114 I=0
IF (FOFX(A))1,2,3
1 XB1=X
IF(LX.NE.0)GO TO 1001
X=X+DELTX
IF(X-B)1000,1000,1004
1004 X=B
LX=1
1000 IF (FOFX(X))1,2,4
4 XB=X
X=X-DELTX/(2.**(I+1))
999 I=I+1
IF(MAXI.LT.I)GO TO 444
IF (FOFX(X))6,2,7
6 L=1
XX=XB
GO TO 18
7 L=2
XX=XB1
GO TO 18
3 XB1=X
IF(KX.NE.0)GO TO 1001
X=X+DELTX
IF(X-B)1002,1002,1003
1003 X=B
KX=1
1002 IF(FOFX(X))5,2,3
5 XB=X
X=X-DELTX/(2.**(I+1))
998 I=I+1
IF(MAXI.LT.I)GO TO 444
IF(FOFX(X))8,2,9
9 L=3
XX=XB
GO TO 18

```

ITR2 2568000000
 ITR2 2569000000
 ITR2 2570000000
 ITR2 2571000000
 ITR2 2572000000
 ITR2 2573000000
 ITR2 2574000000
 ITR2 2575000000
 ITR2 2576000000
 ITR2 2577000000
 ITR2 2578000000
 ITR2 2579000000
 ITR2 2580000000
 ITR2 2581000000
 ITR2 2582000000
 ITR2 2583000000
 ITR2 2584000000
 ITR2 2585000000
 ITR2 2586000000
 ITR2 2587000000
 ITR2 2588000000
 ITR2 2589000000
 ITR2 2590000000
 ITR2 2591000000
 ITR2 2592000000
 ITR2 2593000000
 ITR2 2594000000
 ITR2 2595000000
 ITR2 2596000000
 ITR2 2597000000
 ITR2 2598000000
 ITR2 2599000000
 ITR2 2600000000
 ITR2 2601000000
 ITR2 2602000000
 ITR2 2603000000
 ITR2 2604000000
 ITR2 2605000000
 ITR2 2606000000
 ITR2 2607000000
 ITR2 2608000000
 ITR2 2609000000

APPENDIX C - Continued

```

      261000000
      261100000
      261200000
      261300000
      261400000
      261500000
      261600000
      261700000
      261800000
      261900000
      262000000
      262100000
      262200000
      262300000
      262400000
      262500000
      262600000
      262700000
      262800000
      262900000
      263000000

      ITR2
      ITR2
      ITR2
      ITR2
      ITR2
      ITR2
      ITR2
      ITR2
      ITR2
      ITR2
      ITR2
      ITR2
      ITR2
      ITR2
      ITR2
      ITR2
      ITR2
      ITR2
      ITR2
      ITR2
      ITR2
      ITR2

```

```

      8 L=4
      XX=XB1
      18 IF (ABS(X)-E1)36,36,37
      37 IF (ABS((XX-X)/X)-E1)2,2,17
      36 IF (ABS(XX-X)-E2)2,2,17
      17 GO TO (81,4,81,5),L
      81 XB1=X
      X=X+DELTX/(2.*(I+1))
      GO TO (999,4,998,5),L
      111 ICODE=2
      GO TO 79
      113 ICODE=4
      GO TO 79
      1001 ICODE=3
      GO TO 79
      444 ICODE=1
      GO TO 79
      2 ICODE=0
      79 CONTINUE
      RETURN
      END

```

APPENDIX C - Continued

```

SUBROUTINE SIMEQ(A,N,B,M,DETERM,IPIVOT,NMAX,ISCALE)
C SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS
C ***** DOCUMENT DATE 08-01-68 SUBROUTINE REVISED 08-01-68 *****
C
DIMENSION IPIVOT(N),A(NMAX,N),E(NMAX,M)
EQUIVALENCE (IROW,JROW),(ICOL,JCOL),(AMAX,T,SWAP)
C
C INITIALIZATION
C
5 ISCALE=0
6 R1=10.C*100
7 R2=1.0/R1
10 DETERM=1.0
15 DO 20 J=1,N
20 IPIVOT(J)=0
30 DO 550 I=1,N
C
C SEARCH FOR PIVOT ELEMENT
C
40 AMAX=0.0
45 DO 105 J=1,N
50 IF (IPIVOT(J)-1) 60,105,50
60 DO 100 K=1,N
70 IF (IPIVOT(K)-1) 80,100,740
80 IF (ABS(AMAX)-ABS(A(J,K))) 85,100,100
85 IROW=J
90 ICOL=K
95 AMAX=A(J,K)
100 CONTINUE
105 CONTINUE
106 DETERM=0.0
ISCALE=0
GO TO 740
110 IPIVOT(ICOL)=IPIVOT(ICOL)+1
C
C INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
130 IF (IROW-ICOL) 140,260,140
140 DETERM=-DETERM
150 DO 200 L=1,N
160 SWAP=A(IROW,L)

```

SIMEQ 263100000
 SIMEQ 263200000
 SIMEQ 263300000
 SIMEQ 263400000
 SIMEQ 263500000
 SIMEQ 263600000
 SIMEQ 263700000
 SIMEQ 263800000
 SIMEQ 263900000
 SIMEQ 264000000
 SIMEQ 264100000
 SIMEQ 264200000
 SIMEQ 264300000
 SIMEQ 264400000
 SIMEQ 264500000
 SIMEQ 264600000
 SIMEQ 264700000
 SIMEQ 264800000
 SIMEQ 264900000
 SIMEQ 265000000
 SIMEQ 265100000
 SIMEQ 265200000
 SIMEQ 265300000
 SIMEQ 265400000
 SIMEQ 265500000
 SIMEQ 265600000
 SIMEQ 265700000
 SIMEQ 265800000
 SIMEQ 265900000
 SIMEQ 266000000
 SIMEQ 266100000
 SIMEQ 266200000
 SIMEQ 266300000
 SIMEQ 266400000
 SIMEQ 266500000
 SIMEQ 266600000
 SIMEQ 266700000
 SIMEQ 266800000
 SIMEQ 266900000
 SIMEQ 267000000
 SIMEQ 267100000
 SIMEQ 267200000

APPENDIX C - Continued

```

110
170 A(IROW,L)=A(ICOLU,M,L)
200 A(ICCLU,M,L)=SWAP
205 IF (M) 260,260,210
210 DO 250 L=1,M
220 SWAP=B(IROW,L)
230 B(IROW,L)=B(ICOLU,M,L)
250 B(ICOLU,M,L)=SWAP
260 PIVOT=A(ICOLU,ICOLU)
    IF (PIVOT) 1000,106,1000
C
C    SCALE THE DETERMINANT
C
1000 PIVOTI=PIVOT
1005 IF (ABS(ETERM)-R1) 1030,1010,1010
1010 DETERM=ETERM/R1
    ISCALE=ISCALE+1
    IF (ABS(ETERM)-R1) 1060,1020,1020
1020 DETERM=ETERM/R1
    ISCALE=ISCALE+1
    GO TO 1060
1030 IF (ABS(ETERM)-R2) 1040,1040,1060
1040 DETERM=ETERM*R1
    ISCALE=ISCALE-1
    IF (ABS(ETERM)-R2) 1050,1050,1060
1050 DETERM=ETERM*R1
    ISCALE=ISCALE-1
1060 IF (ABS(PIVOTI)-R1) 1090,1070,1070
1070 PIVOTI=PIVOTI/R1
    ISCALE=ISCALE+1
    IF (ABS(PIVOTI)-R1) 320,1080,1080
1080 PIVOTI=PIVOTI/R1
    ISCALE=ISCALE+1
    GO TO 320
1090 IF (ABS(PIVOTI)-R2) 2000,2000,320
2000 PIVOTI=PIVOTI*R1
    ISCALE=ISCALE-1
    IF (ABS(PIVOTI)-R2) 2010,2010,320
2010 PIVOTI=PIVOTI*R1
    ISCALE=ISCALE-1
320 DETERM=ETERM*PIVOTI
C
C    DIVIDE PIVOT ROW BY PIVOT ELEMENT
C

```


APPENDIX C - Continued

```

340 DO 351 L=1,N
341 IF (IPIVOT(L)-1) 350,351,740
350 A(ICOLU,L)=A(ICOLU,L)/PIVOT
351 CONTINUE
355 IF (M) 380,380,360
360 DO 370 L=1,M
370 B(ICOLU,L)=B(ICOLU,L)/PIVOT
C
C      REDUCE NON-PIVOT ROWS
C
380 DO 550 L1=1,N
390 IF (L1-ICOLU) 400,550,400
400 T=A(L1,ICOLU)
430 DO 451 L=1,N
431 IF (IPIVOT(L)-1) 450,451,740
450 A(L1,L)=A(L1,L)-A(ICOLU,L)*T
451 CONTINUE
455 IF (M) 550,550,460
460 DO 500 L=1,M
500 B(L1,L)=B(L1,L)-B(ICOLU,L)*T
550 CONTINUE
740 RETURN
      END
SIMEQ 271600000
SIMEQ 271700000
SIMEQ 271800000
SIMEQ 271900000
SIMEQ 272000000
SIMEQ 272100000
SIMEQ 272200000
SIMEQ 272300000
SIMEQ 272400000
SIMEQ 272500000
SIMEQ 272600000
SIMEQ 272700000
SIMEQ 272800000
SIMEQ 272900000
SIMEQ 273000000
SIMEQ 273100000
SIMEQ 273200000
SIMEQ 273300000
SIMEQ 273400000
SIMEQ 273500000
SIMEQ 273600000
SIMEQ 273700000
SIMEQ 273800000

```

The following is a listing of input data and a printout from a computer run used to determine the flutter - solution neighborhood:

```

TITANIUM
$OPTION      IOPT=1, IPRINT=2,      ISNDWCH=1, ITHICK=2$
$WING ANG=50.5,RHOW=4.14E-4, E=1.64E7, MU=.3,
      AREA=5.65808E5, DR=3.E-2,
AR=2.5538,
      WTF = 150000.,
DC(1) = 9*0., 3*1.E-6, 1*0., 3*1.E-6,      NC=16,
C(1)=.02, 8*0., .23, 6*0.$
$AIR A=1.2204E4, RHOA=5.14E-8, CORALT=2.5E4, IHP=1$
$CASE NN=6, NCS=1$
$FLUTER
OMINIT=0., DELOM=4., OMFIN=40.,      VINIT=0., DELV=1.E4, VFIN=1.E5$
$OPTIMUM R=300.,RDC=15.,VCR=2.5E4$
$STRSS PLOAD=1., YCR=1.25E5$
$GAGE TBARMIN=.02 $

```

DETERMINATION OF THICKNESS DISTRIBUTION FOR MINIMUM WEIGHT WING
SATISFYING PRESCRIBED FLUTTER AND STRENGTH CRITERIA.
REFERENCE - NACA TN-3440, STEIN-SANDERS, JUNE, 1956.
DEXTER-STROUD RDC288 MAY, 1965.

WING DESCRIPTION

AIR PROPERTIES

FUEL PROPERTIES

EQUATION FOR THICKNESS - TBAR (INCHES)

WHERE $A = 1 - Y/SS$ AND $\theta = 1 - 2 * X / (LE + TE)$

[illegible]

```

XXX LEADING EDGE = A+B*Y
XXX A = -1.18122475E+03
XXX B = 1.21309700E+00

```

DESIGN VARIABLES	FINITE DIFFERENCE INCREMENTS	ANALYTICAL DERIVATIVES	* INDICATES ACTIVE DESIGN VARIABLES
C(1) = 2.0000000E-02	DC(1) = 0.	AD(1) = 1.80836763E+05	
C(2) = 0.	DC(2) = 0.	AD(2) = 1.13761448E+05	
C(3) = 0.	DC(3) = 0.	AD(3) = 8.36219870E+04	
C(4) = 0.	DC(4) = 0.	AD(4) = 6.62179501E+04	
C(5) = 0.	DC(5) = 0.	AD(5) = 0.	
C(6) = 0.	DC(6) = 0.	AD(6) = 0.	
C(7) = 0.	DC(7) = 0.	AD(7) = 0.	
C(8) = 0.	DC(8) = 0.	AD(8) = 0.	
C(9) = 0.	DC(9) = 0.	AD(9) = 6.02789211E+04	
* C(10) = 2.30000000E-01	DC(10) = 1.00000000E-06	AD(10) = 7.58409650E+04	
* C(11) = 0.	DC(11) = 1.00000000E-06	AD(11) = 5.57479913E+04	
* C(12) = 0.	DC(12) = 1.00000000E-06	AD(12) = 4.41453001E+04	
* C(13) = 0.	DC(13) = 0.	AD(13) = 0.	
* C(14) = 0.	DC(14) = 1.00000000E-06	AD(14) = 0.	
* C(15) = 0.	DC(15) = 1.00000000E-06	AD(15) = 0.	
* C(16) = 0.	DC(16) = 1.00000000E-06	AD(16) = 0.	

STATIONS

NUMBER OF SEMISPAN STATIONS.....NN = 6
NUMBER OF CHORDWISE STATIONS = (10 X NCS)...NCS = 1

INPUT OPTIONS

IOPT = 1 1 - ITERATE CN OMEGA AND V, 2 - COMPLEX SEARCH
IPRINT = 2 0 - MATRIX CONSTRUCTION, 1 - SHORT PRINTOUT, 2 - INTERMEDIATE PRINTOUT, 3 - EXTENDED PRINTOUT
ISNDWCH = 1 0 - SGLD WING, 1 - SANDWICH CONSTRUCTION
IHP = 1 0 - THICKNESS VARIATION EFFECTS NOT ACCOUNTED FOR IN PISTON THEORY, 1 - THICKNESS EFFECTS ARE USED

INITIAL WEIGHT = 2.10601572E+C4 PCUNDS

RANGE AND INCREMENT OF ITERATION

OMEGA INITIAL.....CMINIT = 0. RADIAN/SEC.
OMEGA INCREMENT.....DELCH = 4.00000000E+00 RADIAN/SEC.
OMEGA FINAL.....CMFIN = 4.00000000E+01 RADIAN/SEC.

APPENDIX C - Continued

V INITIAL.....VINIT = 0. IN./SEC.
V INCREMENT.....DELV = 1.00000000E+04 IN./SEC.
V FINAL.....VFIN = 1.00000000E+05 IN./SEC.

OMEGA RADIANS/SEC.	V INCHES/SEC.	DETERMINANT	
		REAL PART	IMAGINARY PART
0.	0.	1.35385820E+03	0.
0.	1.00000000E+04	6.23527272E+C3	0.
0.	2.00000000E+04	1.75389121E+04	0.
0.	3.00000000E+04	3.85624065E+04	0.
0.	4.00000000E+04	7.45373284E+C4	0.
0.	5.00000000E+04	1.28062608E+05	0.
0.	6.00000000E+04	2.02432381E+05	0.
0.	7.00000000E+04	2.98951660E+05	0.
0.	8.00000000E+04	4.16746162E+05	0.
0.	9.00000000E+04	5.52370354E+C5	0.
0.	1.00000000E+05	6.95700058E+05	0.
0.	1.00000000E+05	8.70789548E+05	4.04729382E+02
0.	1.00000000E+05	4.86932933E+03	1.06019211E+03
0.	1.00000000E+05	1.46321133E+C4	2.20877611E+03
0.	1.00000000E+05	3.37252092E+C4	3.99144196E+03
0.	1.00000000E+05	6.61215526E+C4	6.51589754E+03
0.	1.00000000E+05	1.15649561E+05	9.83249378E+03
0.	1.00000000E+05	1.85321974E+05	1.39143444E+04
0.	1.00000000E+05	2.76639463E+05	1.86451088E+04
0.	1.00000000E+05	3.88978936E+C5	2.38170344E+04
0.	1.00000000E+05	5.19175443E+05	2.91406108E+04
0.	1.00000000E+05	6.61388855E+C5	3.42657030E+04
0.	1.00000000E+05	-1.19012943E+02	4.13508408E+02
0.	1.00000000E+05	1.79992767E+03	1.28275018E+03
0.	1.00000000E+05	7.77912174E+C3	2.93147930E+03
0.	1.00000000E+05	2.09783986E+C4	5.63686043E+03
0.	1.00000000E+05	4.51457143E+C4	9.63421177E+03
0.	1.00000000E+05	8.41183787E+04	1.50683100E+04
0.	1.00000000E+05	1.41167232E+C5	2.19496329E+04
0.	1.00000000E+05	2.18278806E+C5	3.01231128E+04
0.	1.00000000E+05	3.15492982E+C5	3.92557130E+04
0.	1.00000000E+05	4.30416864E+05	4.88468643E+04
0.	1.00000000E+05	5.58018816E+C5	5.82628045E+04
0.	1.00000000E+05	-7.14752485E+02	4.79753089E+01
0.	1.00000000E+05	-7.97551179E+C2	5.91713881E+02
0.	1.00000000E+05	1.10992291E+03	1.88839878E+03
0.	1.00000000E+05	7.48592875E+C3	4.32166428E+03
0.	1.00000000E+05	2.15900656E+C4	8.26236689E+03
0.	1.00000000E+05	4.70676864E+04	1.39983037E+04
0.	1.00000000E+05	8.73517251E+04	2.16636246E+04
0.	1.00000000E+05	1.44949811E+05	3.11801858E+04

APPENDIX C - Continued

1.23000000E+01	8.00000000E+04	2.20137612E+C5	4.22222806E+04
1.20000000E+01	5.00000000E+04	3.13390870E+C5	5.42134565E+04
1.20000000E+01	1.00000000E+05	4.19076310E+C5	6.63598294E+04
1.60000000E+01	C.	-5.01863233E+02	-2.83643164E+02
1.60000000E+01	1.00000000E+04	-1.48038786E+02	-2.94360145E+02
1.60000000E+01	2.00000000E+04	-2.14373506E+C3	1.56524418E+02
1.60000000E+01	3.00000000E+04	-8.91236506E+C2	1.48655851E+03
1.60000000E+01	4.00000000E+04	4.75239999E+C2	4.16958818E+03
1.60000000E+01	5.00000000E+04	1.789651130E+04	8.61192904E+03
1.60000000E+01	6.00000000E+04	4.18079536E+04	1.51457286E+04
1.60000000E+01	7.00000000E+04	7.92650575E+C4	2.38546356E+04
1.60000000E+01	8.00000000E+04	1.31853513E+C5	3.45457539E+04
1.60000000E+01	5.00000000E+04	1.99339526E+C5	4.67301725E+04
1.60000000E+01	1.00000000E+05	2.79253973E+C5	5.95983440E+04
2.00000000E+01	C.	6.08309797E+C1	-2.41557542E+02
2.00000000E+01	1.00000000E+04	-6.36318151E+02	-5.86991329E+02
2.00000000E+01	2.00000000E+04	-1.82588813E+C3	-8.48499055E+02
2.00000000E+01	3.00000000E+04	-2.79018775E+C3	-6.79007427E+02
2.00000000E+01	4.00000000E+04	-2.01593220E+C3	3.82911785E+02
2.00000000E+01	5.00000000E+04	2.74663557E+C3	2.84124228E+03
2.00000000E+01	6.00000000E+04	1.41866286E+C4	7.14377957E+03
2.00000000E+01	7.00000000E+04	3.49270802E+C4	1.35735029E+04
2.00000000E+01	8.00000000E+04	6.68953752E+C4	2.21509386E+04
2.00000000E+01	5.00000000E+04	1.10692700E+C5	3.25684783E+04
2.00000000E+01	1.00000000E+05	1.65100610E+C5	4.41737555E+04
2.40000000E+01	C.	2.22244419E+C2	5.81439048E+01
2.40000000E+01	1.00000000E+04	1.90826511E+02	-1.94965024E+02
2.40000000E+01	2.00000000E+04	-3.03403039E+C2	-6.17760002E+02
2.40000000E+01	3.00000000E+04	-1.18840698E+C3	-1.02377214E+03
2.40000000E+01	4.00000000E+04	-1.83821707E+03	-1.07328916E+03
2.40000000E+01	5.00000000E+04	-9.74571706E+02	-3.1451754E+02
2.40000000E+01	6.00000000E+04	3.23205619E+C3	1.73705643E+03
2.40000000E+01	7.00000000E+04	1.28466361E+04	5.48698453E+03
2.40000000E+01	8.00000000E+04	2.56814460E+C4	1.11506834E+04
2.40000000E+01	5.00000000E+04	5.47306091E+C4	1.86560189E+04
2.40000000E+01	1.00000000E+05	8.76586555E+04	2.75896026E+04
2.80000000E+01	C.	-1.49034195E+C2	1.76655502E+02
2.80000000E+01	1.00000000E+04	5.43066343E+00	2.43927180E+02
2.80000000E+01	2.00000000E+04	1.02684455E+C2	1.27601535E+02
2.80000000E+01	3.00000000E+04	-3.12764635E+01	-1.66054056E+02
2.80000000E+01	4.00000000E+04	-3.23981193E+02	-4.91181548E+02
2.80000000E+01	5.00000000E+04	-2.78560047E+02	-5.58767000E+02
2.80000000E+01	6.00000000E+04	1.07663911E+03	2.12405013E+01
2.80000000E+01	7.00000000E+04	5.05599517E+03	1.65669934E+03
2.80000000E+01	8.00000000E+04	1.29996066E+04	4.66463035E+03
2.80000000E+01	5.00000000E+04	2.58286587E+C4	9.16118626E+03
2.80000000E+01	1.00000000E+05	4.35819049E+C4	1.49777669E+04
3.20000000E+01	C.	-3.96109287E+02	-9.71373469E+01
3.20000000E+01	1.00000000E+04	-6.84970209E+02	1.18142987E+02
3.20000000E+01	2.00000000E+04	-8.05485331E+C2	3.18596078E+02
3.20000000E+01	3.00000000E+04	-8.13604559E+C2	4.04530416E+02

FLUTTER

APPENDIX C - Continued

3.20000000E+01	4.00000000E+04	-7.52520028E+C2	3.51454508E+02
3.20000000E+01	5.00000000E+04	-4.98960257E+02	2.52895420E+02
3.20000000E+01	6.00000000E+04	2.43573681E+02	3.24532041E+02
3.20000000E+01	7.00000000E+04	3.43079604E+03	8.62756790E+02
3.20000000E+01	8.00000000E+04	6.50737357E+03	2.16383142E+03
3.20000000E+01	9.00000000E+04	1.31024649E+04	4.42216088E+03
3.20000000E+01	1.00000000E+05	2.21725059E+04	7.63473682E+03
3.60000000E+01	C.	5.03155894E+C1	-4.07285573E+02
3.60000000E+01	1.00000000E+04	-6.84461920E+02	-4.11851110E+02
3.60000000E+01	2.00000000E+04	-1.45734639E+03	-2.73275553E+02
3.60000000E+01	3.00000000E+04	-2.07598879E+03	-6.84292707E+01
3.60000000E+01	4.00000000E+04	-2.39251515E+C3	1.19420861E+02
3.60000000E+01	5.00000000E+04	-2.25332479E+03	2.57873902E+02
3.60000000E+01	6.00000000E+04	-1.43901010E+03	4.02128330E+02
3.60000000E+01	7.00000000E+04	3.41648663E+02	6.94474603E+02
3.60000000E+01	8.00000000E+04	3.35962218E+03	1.32188871E+03
3.60000000E+01	9.00000000E+04	7.67705981E+03	2.44137626E+03
3.60000000E+01	1.00000000E+05	1.29187955E+04	4.09511989E+03
4.00000000E+01	C.	7.39313282E+C2	-2.05449836E+02
4.00000000E+01	1.00000000E+04	2.16736263E+02	-5.59688366E+02
4.00000000E+01	2.00000000E+04	-7.54123908E+02	-8.03587676E+02
4.00000000E+01	3.00000000E+04	-1.88553473E+C3	-8.98346482E+02
4.00000000E+01	4.00000000E+04	-2.82052667E+03	-8.57130469E+02
4.00000000E+01	5.00000000E+04	-3.19596339E+C3	-7.11584212E+02
4.00000000E+01	6.00000000E+04	-2.68666611E+03	-4.70395359E+02
4.00000000E+01	7.00000000E+04	-1.06033155E+C3	-9.92404701E+01
4.00000000E+01	8.00000000E+04	1.73106323E+C3	4.68329372E+02
4.00000000E+01	9.00000000E+04	5.42576759E+C3	1.28941757E+C3
4.00000000E+01	1.00000000E+05	9.33476850E+C3	2.35401225E+03

FLUTTER

DETERMINATION OF THICKNESS DISTRIBUTION FOR MINIMUM WEIGHT WING
SATISFYING PRESCRIBED FLUTTER AND STRENGTH CRITERIA.
REFERENCE - NACA TN-3640, STEIN-SANDERS, JUNE, 1956.
DEXTER-STROUD RD6288 MAY, 1969.

DATE 04/27/71

WING DESCRIPTION

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ASPECT RATIO.....AR = 2.55380000E+00
SURFACE AREA.....AREA = 5.65808300E+05 IN.**2
L.E. SWEEP ANGLE.....ANG = 5.05000000E+01 DEGREES
SEMISPAN.....SS = 8.45988374E+02 IN.
ROOT CHORD.....RC = 1.18122475E+03 IN.
DEPTH RATIO.....DR = 3.00000000E-02
WING MATERIAL.....TITANIUM
MASS DENSITY.....RHO = 4.14000000E-04 (LB.*SEC.**2)/(IN.**4)
YOUNGS MODULUS.....E = 1.64000000E+07 PSI
POISSONS RATIO.....MU = 3.00000000E-01

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AIR PROPERTIES

```

SPEED CF. SOUND.....A = 1.22C40000E+04 IN./SEC. = 1.0170000E+03 FT./SEC.
MASS DENSITY.....RHU = 5.1400000E+08 (LB./SEC.**2)/(IN.**4)
CORRESPONDING ALTITUDE... = 2.5000000E+04 FT.

```

FUEL PROPERTIES

```
WEIGHT OF FUEL.....WTF = 1.50000000E+05 LB.
VOLUME OF WING.....VOL = 9.03896471E+06 IN.**3
DENSITY OF FUEL.....RHOF = 4.29517638E-05 (LB.*SEC.**2)/(IN.**4)
```

EQUATION FOR THICKNESS - TBAR (INCHES)

$$\begin{aligned} \text{TBAR} = & C(1) + C(2)*A + C(3)*A**2 + C(4)*A**3 \\ & + (C(5) + C(6)*A + C(7)*A**2 + C(8)*A**3)*B \\ & + (C(9) + C(10)*A + C(11)*A**2 + C(12)*A**3)*(1-B)*B \\ & + (C(13) + C(14)*A + C(15)*A**2 + C(16)*A**3)*(1-3*B)*B \end{aligned}$$

WHERE $A = 1 - Y/SS$ AND $B = 1 - 2 * X / (LE + TE)$

(ITNICK = 2)

[illegible]

APPENDIX C - Continued

* INDICATES ACTIVE
DESIGN VARIABLES

DESIGN VARIABLES	FINITE DIFFERENCE INCREMENTS	ANALYTICAL DERIVATIVES
C(1) = 2.00000000E-02	DC(1) = 0.	AD(1) = 1.80836763E+05
C(2) = 0.	DC(2) = 0.	AD(2) = 1.13761448E+05
C(3) = 0.	DC(3) = 0.	AD(3) = 8.36219870E+04
C(4) = 0.	DC(4) = 0.	AD(4) = 6.62179501E+04
C(5) = 0.	DC(5) = 0.	AD(5) = 0.
C(6) = 0.	DC(6) = 0.	AD(6) = 0.
C(7) = 0.	DC(7) = 0.	AD(7) = 0.
C(8) = 0.	DC(8) = 0.	AD(8) = 0.
C(9) = 0.	DC(9) = 0.	AD(9) = 6.02789211E+04
* C(10) = 2.30000000E-01	DC(10) = 1.00000000E-06	AD(10) = 7.58409650E+04
* C(11) = 0.	DC(11) = 1.00000000E-06	AD(11) = 5.57479913E+04
* C(12) = 0.	DC(12) = 1.00000000E-06	AD(12) = 4.41453001E+04
* C(13) = 0.	DC(13) = 0.	AD(13) = 0.
* C(14) = 0.	DC(14) = 1.00000000E-06	AD(14) = 0.
* C(15) = 0.	DC(15) = 1.00000000E-06	AD(15) = 0.
* C(16) = 0.	DC(16) = 1.00000000E-06	AD(16) = 0.

STATIONS

NUMBER OF SEMISPAN STATIONS.....NN = 6
NUMBER OF CHORDWISE STATIONS = (10 X NCS)...NCS = 1

INPUT OPTIONS

IOP1 = 2 1 - ITERATE CN OMEGA AND V, 2 - COMPLEX SEARCH
IOP2 = 1 0 - MATRIX CONSTRUCTION, 1 - SHORT PRINTOUT, 2 - INTERMEDIATE PRINTOUT, 3 - EXTENDED PRINTOUT
ISNDWCH = 1 0 - SOLID WING, 1 - SANDWICH CONSTRUCTION
IHP = 1 0 - THICKNESS VARIATION EFFECTS NOT ACCOUNTED FOR IN PISTON THEORY, 1 - THICKNESS EFFECTS ARE USED

INITIAL WEIGHT = 2.10661572E+04 POUNDS

INPUT DATA FOR SEARCH ROUTINE

NUMBER OF DESIGN VARIABLES.....NC = 16
R = 3.00000000E+02
R REDUCTION FACTOR.....RDC = 1.50000000E+01

DESIGN CGNSTRAINTS CONSIDERED

FLUTTER - T
STRESS - T
MINGAGE - T

INPUT DATA FOR FLUTTER

CMEGA INITIAL.....OMINIT = 2.40000000E+01 RADIANS/SEC.
OMEGA INCREMENT.....DELOM = 1.00000000E-04 RADIANS/SEC.

V INITIAL.....VINIT = 5.00000000E+04 IN./SEC. = 4.16666667E+03 FT./SEC.
V INCREMENT.....DELV = 1.00000000E-01 IN./SEC. = 8.33333333E-03 FT./SEC.

V CRITICAL.....VCR = 2.50000000E+04 IN./SEC. = 2.08333333E+03 FT./SEC.

INPUT DATA FOR STRESS

UNIFORM PRESSURE LOADING....PLOAD = 1.00000000E+00 PSI
YIELD STRESS.....YCR = 1.25000000E+05 PSI

INPUT FOR MINGAGE

MINIMUM GAGE.....TBARMIN= 2.00000000E-02 INCHES

CMEGA RADIANS/SEC.	V INCHES/SEC.	REAL PART	IMAGINARY PART	DETERMINANT
2.40000000E+01	5.30000000E+04	-9.74571706E+02	-3.14151754E+02	-3.14151754E+02
2.40001000E+01	5.30000000E+04	-9.74579281E+02	-3.14189575E+02	-3.14189575E+02
2.40000000E+01	5.30001000E+04	-9.74549739E+02	-3.14138482E+02	-3.14138482E+02
2.48261288E+01	5.47214022E+04	1.43529902E+02	5.25500697E+01	5.25500697E+01
2.48262288E+01	5.47214022E+04	1.43503523E+02	5.25087797E+01	5.25087797E+01
2.48261288E+01	5.47215022E+04	1.43561220E+02	5.25661784E+01	5.25661784E+01
2.47453825E+01	5.41984633E+04	2.10816949E+00	7.31002324E-01	7.31002324E-01
2.47454825E+01	5.41984633E+04	2.08451435E+00	6.90427856E-01	6.90427856E-01
2.47453825E+01	5.41985633E+04	2.13828276E+00	7.46726934E-01	7.46726934E-01
2.47480721E+01	5.41904331E+04	4.76938518E-04	1.78202736E-04	1.78202736E-04
2.47481721E+01	5.41904331E+04	-2.31405444E-02	-4.03880499E-02	-4.03880499E-02
2.47480721E+01	5.41905331E+04	3.05730103E-02	1.58975004E-02	1.58975004E-02
2.47480718E+01	5.41904313E+04	-5.58078517E-09	1.20520473E-09	1.20520473E-09

COMPLEX DETERMINANT CONVERGED TO 0

FLUTTER Q = 1.08677798E+C4 LB./SQ.FT.

APPENDIX C - Continued

STRESS TABLE

Y = 0.									
X	-ZU	EP X	EP Y	GAM XY	SIG X	SIG Y	TAU XY	Y	
-6.78539E+02	-1.73257E+01	C.	-1.13250E-03	C.	-6.12296E+03	-2.04099E+04	0.	1.81477E+04	
-8.46581E+02	-1.43903E+01	C.	-3.05253E-04	C.	-1.65038E+03	-5.50127E+03	0.	4.88963E+03	
-9.51880E+02	-9.53362E+00	C.	-9.29141E-06	C.	-5.02349E+01	-1.67450E+02	0.	1.48833E+02	
-1.10153E+03	-4.4591CE+00	C.	1.46444E-05	C.	7.91761E+01	2.63920E+02	0.	2.34577E+02	
-1.16581E+03	-9.12604E-01	C.	1.23489E-06	C.	6.78428E+00	2.26143E+01	0.	2.01000E+01	
-3.02685E+02	-7.3257E+00	C.	-2.34589E-03	C.	-1.26833E+04	-4.22775E+04	0.	3.75771E+04	
-3.34644E+02	-1.4393CE+01	C.	-3.23914E-03	C.	-1.75127E+04	-5.83758E+04	0.	5.18855E+04	
-1.89365E+02	-1.4393CE+01	C.	-3.05825E-03	C.	-1.65347E+04	-5.51158E+04	0.	4.89880E+04	
-7.96952E+01	-4.4591CE+00	0.	-1.79997E-03	C.	-9.73171E+03	-3.24390E+04	0.	2.88325E+04	
-1.54111E+01	-9.12604E-01	C.	-4.16854E-04	C.	-2.25376E+03	-7.51253E+03	0.	6.67728E+03	
Y = 1.41664729E+02									
X	-ZU	EP X	EP Y	GAM XY	SIG X	SIG Y	TAU XY	Y	
-5.79821E+02	-1.48050E+01	-1.17071E-04	-1.53886E-03	-1.47589E-03	-1.04298E+04	-2.83562E+04	-9.30947E+03	2.96238E+04	
-7.23414E+02	-1.22967E+01	-9.72362E-05	-7.64465E-04	-1.00773E-03	-5.88554E+03	-1.43029E+04	-6.35644E+03	1.66270E+04	
-8.47174E+02	-8.15173E+00	-6.44598E-05	-2.23712E-04	-5.43021E-04	-2.37121E+03	-4.38924E+03	-3.42521E+03	7.04406E+03	
-9.41271E+02	-3.81036E+00	-3.01304E-05	-7.98297E-06	-2.09723E-04	-5.86170E+02	-3.06772E+02	-1.32287E+03	2.34688E+03	
-9.96203E+02	-7.79832E-01	-6.16652E-06	9.68813E-06	-3.76306E-05	-5.87531E+01	-1.41259E+02	-2.57362E+02	4.48027E+02	
-4.29551E+02	-1.48050E+01	-1.17071E-04	-2.21350E-03	-1.75070E-03	-1.43773E+04	-4.05246E+04	-1.17429E+04	4.04447E+04	
-2.85957E+02	-1.22967E+01	-9.72362E-05	-2.39570E-03	-1.67221E-03	-1.47350E+04	-4.37010E+04	-1.05478E+04	4.26281E+04	
-6.1757E+02	-8.15173E+00	-6.44598E-05	-1.91893E-03	-1.23356E-03	-1.15366E+04	-3.49314E+04	-7.78093E+03	3.36442E+04	
-6.81066E+01	-3.81036E+00	-3.01304E-05	-1.01690E-03	-6.20704E-04	-6.04100E+03	-1.84899E+04	-3.91521E+03	1.76819E+04	
-1.3169CE+01	-7.79832E-01	-6.16652E-06	-2.22779E-04	-1.32325E-04	-1.31561E+03	-4.04826E+03	-8.34668E+02	3.85789E+03	
Y = 2.83239458E+02									
X	-ZU	EP X	EP Y	GAM XY	SIG X	SIG Y	TAU XY	Y	
-4.81102E+02	-1.22843E+01	-2.14966E-04	-1.62507E-03	-2.37077E-03	-1.26600E+04	-3.04492E+04	-1.49497E+04	3.70462E+04	
-6.00248E+02	-1.02031E+01	-1.78546E-04	-1.01379E-03	-1.83123E-03	-8.69888E+03	-1.92357E+04	-1.15508E+04	2.60533E+04	
-7.0368E+02	-6.76383E+00	-1.18362E-04	-4.67209E-04	-1.13525E-03	-4.65912E+03	-9.05996E+03	-7.16084E+03	1.46769E+04	
-7.81013E+02	-3.16162E+00	-5.53258E-05	-1.42604E-05	-5.82930E-04	-1.76808E+03	-2.86913E+03	-3.17208E+03	6.03915E+03	
-8.26592E+02	-6.47059E-01	-1.13230E-05	-1.98040E-05	-9.59910E-05	-3.11135E+02	-4.18126E+02	-6.28189E+02	1.15168E+03	
-3.56417E+02	-1.22843E+01	-2.14966E-04	-2.01875E-03	-2.54307E-03	-1.47887E+04	-3.75400E+04	-1.60409E+04	4.29530E+04	
-2.37271E+02	-1.02031E+01	-1.78546E-04	-1.96567E-03	-2.24951E-03	-1.38453E+04	-3.63903E+04	-1.41892E+04	4.02011E+04	
-1.34250E+02	-6.76383E+00	-1.18362E-04	-1.45643E-03	-1.33219E-03	-1.00074E+04	-2.68876E+04	-9.90275E+03	2.91240E+04	
-5.6560E+01	-3.16162E+00	-5.53258E-05	-7.31344E-04	-7.81602E-04	-4.95118E+03	-1.34794E+04	-4.80739E+03	1.44466E+04	
-1.09699E+01	-6.47059E-01	-1.11230E-05	-1.55457E-04	-1.59201E-04	-1.04455E+03	-2.86286E+03	-1.00419E+03	3.05317E+03	
Y = 4.24994187E+02									
X	-ZU	EP X	EP Y	GAM XY	SIG X	SIG Y	TAU XY	Y	
-3.82383E+02	-9.76368E+00	-2.33427E-04	-1.51290E-03	-2.71956E-03	-1.27109E+04	-2.96088E+04	-1.71542E+04	3.93025E+04	
-4.77081E+02	-8.10049E+00	-1.53879E-04	-1.00076E-03	-2.20427E-03	-9.39139E+03	-2.07059E+04	-1.39038E+04	3.00434E+04	
-5.58963E+02	-5.37594E+00	-1.28526E-04	-5.94136E-04	-1.42999E-03	-5.52859E+03	-1.14024E+04	-9.01994E+03	1.84859E+04	
-6.20754E+02	-2.51287E+00	-6.00771E-05	-2.30698E-04	-6.57393E-04	-2.33000E+03	-4.48245E+03	-4.14663E+03	8.16461E+03	
-6.56981E+02	-5.14287E-01	-1.22954E-05	-4.1479E-05	-1.33219E-04	-4.45680E+02	-8.13450E+02	-8.40307E+02	1.61765E+03	
-2.83283E+02	-9.76368E+00	-2.33427E-04	-1.83174E-03	-2.78828E-03	-1.41103E+04	-3.44273E+04	-1.75876E+04	4.26396E+04	
-1.88885E+02	-6.10049E+00	-1.53879E-04	-1.71661E-03	-2.37077E-03	-1.27751E+04	-3.19850E+04	-1.49519E+04	3.80556E+04	
-1.06703E+02	-5.37594E+00	-1.28526E-04	-1.24454E-03	-1.60266E-03	-9.04501E+03	-2.31233E+04	-1.01091E+04	2.67199E+04	
-4.9113E+01	-2.51287E+00	-6.00771E-05	-6.17790E-04	-7.60157E-04	-4.42288E+03	-1.14580E+04	-4.79484E+03	1.30039E+04	
-8.68476E+00	-5.14287E-01	-1.22954E-05	-1.30638E-04	-1.56898E-04	-9.27897E+02	-2.40384E+03	-9.89662E+02	2.72277E+03	

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APPENDIX C - Continued

THE FOLLOWING INFORMATION IS FOR THE FINAL DESIGN

CTEST = 7.77971542E-03

ESTIMATED MINIMUM WEIGHT LESS THAN 2 PERCENT LOWER THAN PRESENT WEIGHT
DESIGN CONSIDERED CONVERGED

FINAL WEIGHT= 8.13621051E+03

FINAL CONSTANTS		DETERMINANT	
OMEGA RADIANS/SEC.	V INCHES/SEC.	REAL PART	IMAGINARY PART
2.00000000E-02	0.	0.	0.
0.	0.	0.	0.
0.	3.43637754E-03	1.49277271E-01	-9.20381315E-02
0.	7.56679026E-02	-4.68304085E-01	4.50574029E-01
1.59740137E+01	2.50952961E+04	9.45799314E-17	7.84263020E-17
1.59741137E+01	2.50952961E+04	-1.72088298E-09	-1.15412245E-08
1.59740137E+01	2.50953961E+04	5.56490046E-05	5.55167102E-09
1.59740137E+01	2.50952961E+04	-9.11912192E-17	4.76074509E-17

COMPLEX DETERMINANT CONVERGED TO 0

FLUTTER Q = 2.33066720E+03 LB./SQ.FT.

STRESS TABLE

Y = 0.

X	-ZU	EP X	EP Y	GAM XY	SIG X	SIG Y	TAU XY	Y
-6.78539E+02	-1.73257E+01	0.	-3.53132E-03	C.	-1.90924E+04	-6.36415E+04	0.	5.65658E+04
-8.46581E+02	-1.43903E+01	0.	-1.30425E-03	C.	-7.05154E+03	-2.53505E+04	0.	2.08918E+04
-9.51880E+02	-9.53962E+00	0.	-1.87883E-04	C.	-1.01581E+03	-3.38602E+03	0.	3.00956E+03
-1.10153E+03	-4.45910E+00	0.	7.71594E-05	C.	4.17169E+02	1.39056E+03	0.	1.23596E+03
-1.16581E+03	-9.12604E-01	0.	2.9516E-05	C.	1.59773E+02	5.32578E+02	0.	4.73365E+02
-5.02685E+02	-1.73257E+01	0.	-6.20327E-03	C.	-3.35386E+04	-1.111795E+05	0.	9.93658E+04
-3.34644E+02	-1.43903E+01	0.	-7.76484E-03	C.	-4.19813E+04	-1.39938E+05	0.	1.24379E+05
-1.89345E+02	-9.53962E+00	0.	-6.90187E-03	C.	-3.73156E+04	-1.24385E+05	0.	1.10556E+05
-7.96952E+01	-4.45910E+00	0.	-3.91873E-03	C.	-2.11870E+04	-7.06232E+04	0.	6.27713E+04
-1.54111E+01	-9.12604E-01	0.	-8.91147E-04	C.	-4.81807E+03	-1.60602E+04	0.	1.42746E+04

Y = 1.41664729E+02

X	-ZU	EP X	EP Y	GAM XY	SIG X	SIG Y	TAU XY	Y
-5.79821E+02	-1.48050E+01	-1.75742E-04	-4.27863E-03	-3.56247E-03	-2.63000E+04	-7.80596E+04	-2.24710E+04	7.90370E+04
-7.23414E+02	-1.22967E+01	-1.45967E-04	-2.21393E-03	-2.48865E-03	-1.46004E+04	-4.06886E+04	-1.56976E+04	4.48766E+04
-8.47574E+02	-8.15173E+00	-9.67646E-05	-7.94129E-04	-1.38023E-03	-6.03739E+03	-1.48348E+04	-8.70604E+03	1.98581E+04
-9.41271E+02	-3.81036E+00	-4.52306E-05	-1.60717E-04	-5.50075E-04	-1.58408E+03	-3.14099E+03	-3.46971E+03	6.59764E+03
-9.96203E+02	-7.79832E-01	-9.25695E-06	-9.85667E-06	-1.01170E-04	-2.20119E+02	-2.27685E+02	-6.38151E+02	1.12778E+03
-4.29551E+02	-1.48050E+01	-1.75742E-04	-6.19456E-03	-4.15498E-03	-3.86587E+04	-1.12588E+05	-2.62083E+04	1.09331E+05
-2.85957E+02	-1.22967E+01	-1.45967E-04	-6.84652E-03	-3.92128E-03	-3.96470E+04	-1.24177E+05	-2.47343E+04	1.17915E+05
-1.61757E+02	-8.15173E+00	-9.67646E-05	-5.60842E-03	-2.86905E-03	-3.20663E+04	-1.01598E+05	-1.80971E+04	9.52630E+04
-6.81006E+01	-3.81036E+00	-4.52306E-05	-3.02598E-03	-1.43616E-03	-1.71754E+04	-5.47788E+04	-9.05887E+03	5.10095E+04
-1.31690E+01	-7.79832E-01	-9.25695E-06	-6.70047E-04	-3.05335E-04	-3.78950E+03	-1.21256E+04	-1.92596E+03	1.12593E+04

APPENDIX C – Continued

Y = 2.8329458E+02	EP X	EP Y	GAM XY	SIG X	SIG Y	TAU XY	Y
-4.81102E+02	-4.63473E-04	-4.44427E-03	-6.25005E-03	-3.23811E+04	-8.26004E+04	-3.94234E+04	9.92939E+04
-6.00248E+02	-3.8450E-04	-2.71032E-03	-4.66743E-03	-2.16236E+04	-3.10348E+04	-2.94407E+04	6.75932E+04
-7.30268E+02	-2.55191E-04	-1.27777E-03	-2.79393E-03	-1.15074E+04	-2.44077E+04	-1.76233E+04	3.71952E+04
-8.10133E+02	-1.19284E-04	-4.23116E-04	-1.20007E-03	-4.43735E+03	-8.27312E+03	-7.56969E+03	1.49429E+04
-8.26592E+02	-2.44128E-05	-6.65361E-05	-2.32902E-04	-7.99741E+02	-1.33124E+03	-1.46908E+03	2.79673E+03
-3.56417E+02	-4.63473E-04	-5.75938E-03	-6.90992E-03	-3.94913E+04	-1.06301E+05	-4.35857E+04	1.19836E+05
-2.37271E+02	-1.20311E+01	-5.89617E-03	-6.26295E-03	-3.88157E+04	-1.08342E+05	-3.95047E+04	1.17137E+05
-1.34250E+02	-2.55191E-04	-4.58233E-03	-4.45203E-03	-2.93739E+04	-3.36224E+04	-2.80820E+04	8.83856E+04
-5.65060E+01	-1.19284E-04	-2.53988E-03	-2.18690E-03	-1.50707E+04	-3.37148E+04	-1.37943E+04	4.52786E+04
-1.09269E+01	-2.44128E-05	-5.19703E-04	-4.60280E-04	-3.242979E+03	-9.49806E+03	-2.90330E+03	9.75680E+03
Y = 4.24954187E+02	EP X	EP Y	GAM XY	SIG X	SIG Y	TAU XY	Y
-3.82383E+02	-7.11790E-04	-4.34126E-03	-7.79762E-03	-3.62993E+04	-8.20864E+04	-4.91850E+04	1.11057E+05
-4.77081E+02	-5.91196E-04	-2.92056E-03	-6.14547E-03	-2.64448E+04	-5.58306E+04	-3.87637E+04	8.27516E+04
-5.98633E+02	-3.91916E-04	-1.52538E-03	-3.88420E-03	-1.53102E+04	-2.96093E+04	-2.50030E+04	4.95841E+04
-6.20754E+02	-1.83193E-04	-5.62574E-04	-1.74865E-03	-6.34527E+03	-1.11363E+04	-1.10300E+04	2.14149E+04
-2.56981E+02	-3.74925E-05	-9.67946E-05	-3.49849E-04	-1.19902E+03	-9.94714E+03	-2.20674E+03	4.18371E+03
-2.83283E+02	-7.11790E-04	-5.61233E-03	-8.21473E-03	-4.07325E+04	-9.68639E+04	-5.18160E+04	1.23089E+05
-1.88585E+02	-5.51156E-04	-4.90319E-03	-7.15401E-03	-3.71641E+04	-9.15615E+04	-4.51253E+04	1.11673E+05
-1.06703E+02	-3.91916E-04	-3.58578E-03	-4.93201E-03	-2.64499E+04	-6.67417E+04	-3.11114E+04	7.93253E+04
-4.49113E+01	-1.83193E-04	-1.78924E-03	-2.37244E-03	-1.29752E+04	-3.32605E+04	-1.49846E+04	3.89045E+04
-8.68476E+00	-3.74925E-05	-3.79340E-04	-4.93577E-04	-2.72662E+03	-7.09916E+03	-3.11333E+03	8.17738E+03
Y = 5.66558916E+02	EP X	EP Y	GAM XY	SIG X	SIG Y	TAU XY	Y
-3.83664E+02	-7.15596E-04	-3.79068E-03	-7.78350E-03	-3.33911E+04	-8.21845E+04	-4.90959E+04	1.05577E+05
-3.53914E+02	-5.4358E-04	-2.75455E-03	-6.49611E-03	-2.56042E+04	-5.28558E+04	-4.09754E+04	8.44567E+04
-4.1457E+02	-3.94011E-04	-1.55101E-03	-4.32434E-03	-1.54865E+04	-3.08225E+04	-2.72766E+04	5.39533E+04
-4.60496E+02	-1.84173E-04	-6.19388E-04	-2.02768E-03	-6.63793E+03	-2.20583E+04	-1.27898E+04	2.4982E+04
-8.87370E+02	-3.76529E-05	-1.11135E-04	-4.15742E-04	-1.28017E+03	-2.20667E+03	-2.62237E+03	4.93090E+03
-2.10148E+02	-7.15596E-04	-4.16819E-03	-7.74403E-03	-3.05332E+04	-8.93880E+04	-4.88471E+04	1.08876E+05
-1.39898E+02	-5.4358E-04	-3.6673E-03	-6.40073E-03	-2.56363E+04	-5.93026E+04	-4.03738E+04	9.22473E+04
-7.91558E+01	-3.94011E-04	-2.49961E-03	-4.22522E-03	-1.66152E+04	-4.71782E+04	-2.66514E+04	6.17177E+04
-3.33167E+01	-1.84173E-04	-1.7841E-03	-1.96865E-03	-9.69031E+03	-2.2329E+04	-1.24177E+04	2.89026E+04
-6.44264E+00	-3.76529E-05	-2.41128E-04	-4.02149E-04	-1.98347E+03	-4.55102E+03	-2.53663E+03	5.90954E+03
Y = 7.0822345E+02	EP X	EP Y	GAM XY	SIG X	SIG Y	TAU XY	Y
-1.84945E+02	-2.94707E-04	-2.03177E-03	-5.68909E-03	-1.62962E+04	-3.82099E+04	-3.58850E+04	7.04705E+04
-2.30748E+02	-2.44777E-04	-1.56758E-03	-5.05592E-03	-1.28866E+04	-2.95742E+04	-3.18912E+04	6.09158E+04
-2.70351E+02	-1.62267E-04	-9.2921E-04	-3.54122E-03	-7.94830E+03	-1.76237E+04	-2.23369E+04	4.15994E+04
-3.00237E+02	-7.56486E-05	-3.8373E-04	-1.72214E-03	-3.44165E+03	-7.3578E+03	-1.08627E+04	1.98569E+04
-3.17759E+02	-1.55235E-05	-2.14958E-05	-3.60477E-04	-6.66308E+02	-1.37424E+03	-2.27378E+03	4.11379E+03
-1.37014E+02	-2.94707E-04	-2.08356E-03	-5.27243E-03	-1.65761E+04	-3.91432E+04	-3.32568E+04	6.69042E+04
-9.12119E+01	-2.44777E-04	-1.65278E-03	-4.04846E-03	-1.35636E+04	-3.18307E+04	-2.55364E+04	5.21705E+04
-5.16086E+01	-1.62267E-04	-1.05934E-03	-2.49425E-03	-8.65180E+03	-1.99687E+04	-1.57330E+04	3.23020E+04
-2.17221E+01	-7.58486E-05	-6.1179E-04	-1.09902E-03	-3.86035E+03	-8.72143E+03	-6.93230E+03	1.41940E+04
-4.20052E+00	-1.55235E-05	-8.93939E-05	-2.16904E-04	-7.62781E+02	-1.69400E+03	-1.36817E+03	2.78836E+03
Y = 8.49988374E+02	EP X	EP Y	GAM XY	SIG X	SIG Y	TAU XY	Y
-8.62267E+01	-3.5621E-04	-1.06986E-04	-2.10527E-03	-5.84958E+03	-2.33458E-08	-1.32794E+04	2.37325E+04
-1.07581E+02	-2.56201E-04	-8.8603E-05	-1.87229E-03	-4.85770E+03	-2.4397E-09	-1.18098E+04	2.10241E+04
-1.26045E+02	-1.21227E+00	-1.96334E-05	-1.31208E-03	-3.22027E+03	1.23724E-08	-8.27623E+03	1.46921E+04
-1.39979E+02	-5.66649E-01	-2.7530E-05	-6.30319E-04	-1.50532E+03	1.30074E-08	-4.02632E+03	7.13439E+03
-1.48148E+02	-1.15971E-01	-1.8745E-05	-5.6354E-04	-3.08055E+02	3.63221E-09	-8.22941E+02	1.49220E+03
-6.3897E+01	-2.20169E+00	-3.5621E-04	-1.94941E-03	-5.84858E+03	-2.67300E-08	-1.22963E+04	2.20862E+04
-2.52535E+01	-1.82868E+00	-2.56201E-04	-1.49543E-03	-4.85770E+03	-1.24193E-08	-9.43273E+03	1.70448E+04
-2.40613E+01	-1.21227E+00	-1.96334E-05	-9.20446E-04	-3.22027E+03	3.88058E-09	-5.80589E+03	1.05991E+04
-1.01274E+01	-5.66649E-01	-9.17834E-05	-4.05232E-04	-1.50525E+03	7.95452E-09	-2.55608E+03	4.67615E+03
-1.95840E+00	-1.15971E-01	-1.8745E-05	-5.63534E-06	-3.08065E+02	2.51815E-09	-5.04200E+02	9.26043E+02

FAILURE RATIO 1 = 9.94202630E-01 G1 = 3.75736952E-03
FAILURE RATIO 2 = 9.00804250E-01 G2 = 9.1957504E-02
FAILURE RATIO 3 = 8.02510251E-01 G3 = 1.97489749E-01

APPENDIX C - Continued

THICKNESS TABLE

Y	X	TEAR
0	-1.18122475E+03	2.CC00000E-02
0	-1.12216351E+03	2.16209764E-02
0	-1.06310228E+03	2.51570862E-02
0	-1.00404104E+03	3.02607C23E-02
0	-9.44979800E+02	3.65841976E-02
0	-8.85918563E+02	4.37799450E-02
0	-8.26857325E+02	5.15003176E-02
0	-7.67796088E+02	5.93976881E-02
0	-7.08734850E+02	6.71244295E-02
0	-6.49673613E+02	7.43329148E-02
0	-5.90612375E+02	8.06755168E-02
0	-5.31551138E+02	8.58046084E-02
0	-4.72489900E+02	8.92725627E-02
0	-4.13428663E+02	9.10317524E-02
0	-3.54367425E+02	9.04345506E-02
0	-2.95306188E+02	8.72333301E-02
0	-2.36244950E+02	8.10804639E-02
0	-1.77183713E+02	7.16283249E-02
0	-1.18122475E+02	5.85292859E-02
0	-5.90612375E+01	4.14357200E-02
0	3.91082722E-11	2.00000000E-02
71	-1.09529822E+03	2.00000000E-02
71	-1.04053331E+03	2.70432003E-02
71	-9.85768399E+02	3.41698956E-02
71	-9.31003488E+02	4.12426052E-02
71	-8.76238577E+02	4.81238484E-02
71	-8.21473666E+02	5.46761444E-02
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71	-5.47649111E+02	7.76515892E-02
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71	-2.19059644E+02	6.51213857E-02
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71	-5.47649111E+01	3.48796036E-02
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APPENDIX C – Continued

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142	-8.07497354E+02	5.46297900E-02
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142	-7.06560185E+02	6.52154375E-02
142	-6.56091600E+02	6.88554697E-02
142	-6.05623015E+02	7.14048865E-02
142	-5.55154431E+02	7.28721224E-02
142	-5.04685846E+02	7.32656117E-02
142	-4.54217261E+02	7.25937888E-02
142	-4.03748677E+02	7.08650880E-02
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APPENDIX C – Continued

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APPENDIX C – Continued

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APPENDIX C – Continued

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637	-1.63154395E+02	2.77649272E-02
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779	-1.06214819E+02	2.15851675E-02
779	-9.44131720E+01	2.18553045E-02
779	-8.26115255E+01	2.20602749E-02
779	-7.08098790E+01	2.21801931E-02

APPENDIX C – Concluded

779	-5.90082325E+01	2.21951731E-02
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779	-2.36032930E+01	2.14116270E-02
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779	2.50111043E-12	2.00000000E-02
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850	-9.00638406E+01	2.00000000E-02
850	-8.25585205E+01	2.00000000E-02
850	-7.50532005E+01	2.00000000E-02
850	-6.75478804E+01	2.00000000E-02
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APPENDIX D

RESULTS FOR TRAPEZOIDAL FLAT PLATE

In order to check the validity of the structural analysis, this appendix presents a comparison of theoretical and experimental natural frequencies of vibration of a cantilevered trapezoidal plate. The theoretical flutter speed and frequency for that plate are also presented but are not compared with experimental results. The dimensions and material properties of the aluminum plate are given in figure 12. The results are summarized in table I where theoretical vibration results are shown from SWIFT, which is the computer program based on the analysis presented in this report and described in appendix C, and from NASTRAN, a general-purpose finite-element structural-analysis computer program. The experimental results and the theoretical results obtained by using NASTRAN are taken from reference 26.

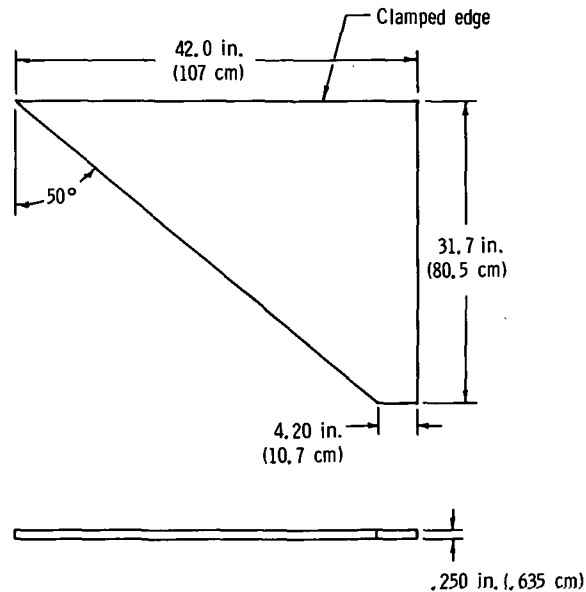


Figure 12.- Dimensions and material properties of aluminum plate used to obtain results given in table I.

$E = 1.00 \times 10^4$ ksi (68.9 GN/m^2); $\mu = 0.30$; Density, 0.100 lbm/in^3 (2770 kg/m^3).

Achieving a true "fixed" boundary condition experimentally is very difficult. The support must be massive, and the specimen-attachment bolts must be numerous, heavy, and well-placed. The experimental results presented in table I are believed to approach a true fixed-boundary condition.

TABLE I.- EXPERIMENTAL AND THEORETICAL NATURAL FREQUENCIES
AND THEORETICAL FLUTTER SPEED AND FREQUENCY OF
CANTILEVERED TRAPEZOIDAL PLATE
SHOWN IN FIGURE 11

Experimental natural frequency, Hz	Theoretical natural frequency, Hz, for –	
	NASTRAN (109 grid points with 327 degrees of freedom)	SWIFT (10 spanwise stations with 33 degrees of freedom)
12.3	12.1	12.1
38.6	41.8	42.9
63.5	63.7	61.4
95.0	96.9	107.0
151.3	147.7	150.2

Theoretical flutter speed 1180 m/s (3890 ft/s)
Theoretical flutter frequency 35.7 Hz

APPENDIX D – Continued

A comparison between the experimental results and both sets of theoretical results indicates that the structural model used in *SWIFT* is adequate for simple wing structures.

In order to provide some insight into the flutter calculations, curves showing the locus of points for which the real part of the flutter determinant is zero and curves showing the locus of points for which the imaginary part of the flutter determinant is zero are shown in figure 13. For this case, the two flutter solutions shown in figure 13 are widely separated.

Following figure 13, a portion of the computer printout which contains the pertinent input data for the flutter calculations is presented. The input data used to obtain the natural frequencies differ only in that RHOA , the density of the air, was set equal to zero.

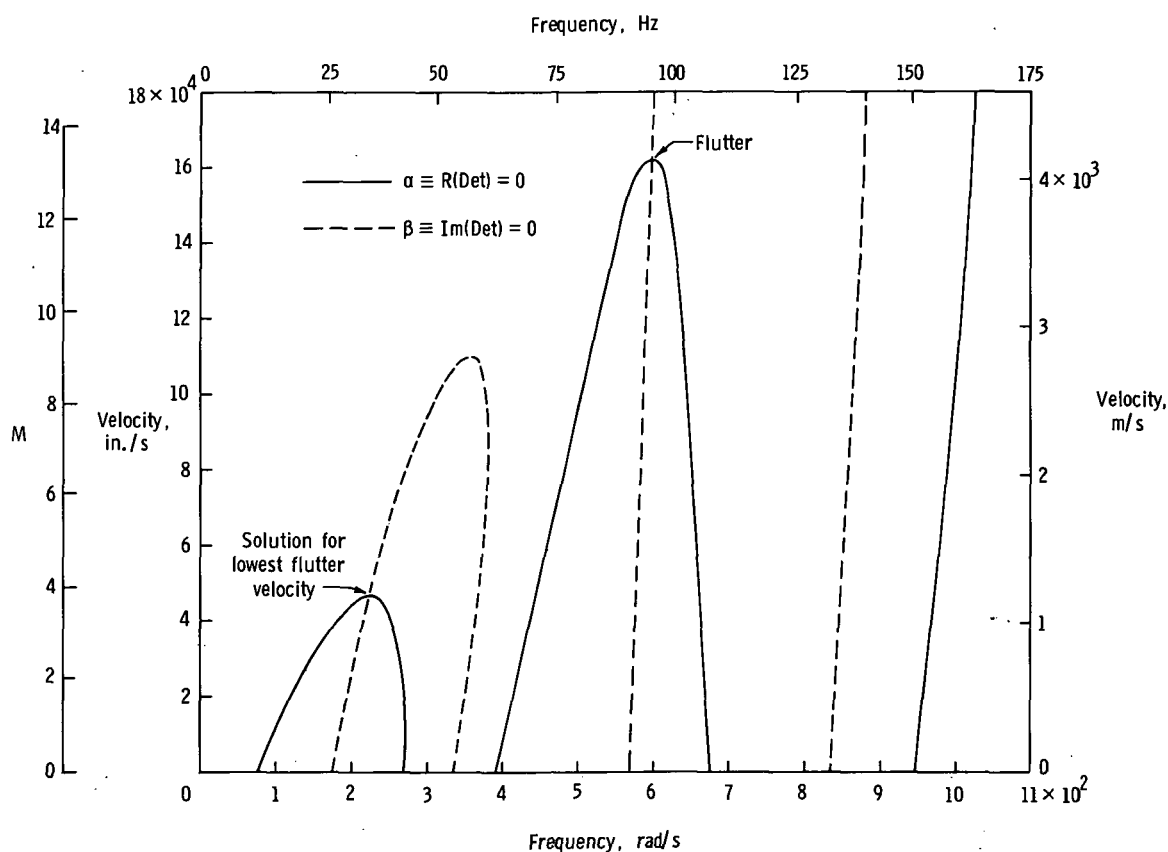


Figure 13.- Locus of points for which the real part of the flutter determinant, $R(\text{Det})$, is equal to zero (solid lines) and for which the imaginary part, $\text{Im}(\text{Det})$, is zero (dashed lines).

* INDICATES ACTIVE DESIGN VARIABLES

STATIONS

NUMBER OF SEMISPAN STATIONS.....NN = 10
NUMBER OF CHORDWISE STATIONS = (10 X NCS)...NCS = 1

INPUT OPTIONS

IOP1 = 1 1 - ITERATE CN GMEGA AND V, 2 - COMPLEX SEARCH
IOP2 = 2 0 - MATRIX CONSTRUCTION, 1 - SHORT PRINTOUT, 2 - INTERMEDIATE PRINTOUT, 3 - EXTENDED PRINTOUT
ISNDWCH = 0 0 - SOLID WING, 1 - SANDWICH CONSTRUCTION
IHP = 0 0 - THICKNESS VARIATION EFFECTS NOT ACCOUNTED FOR IN PISTON THEORY, 1 - THICKNESS EFFECTS ARE USED

INITIAL WEIGHT = 3.65964435E+01 POUNDS

RANGE AND INCREMENT OF ITERATION

CMEGA INITIAL.....CMINIT = 3.20000000E+02 RADIANS/SEC.
CMEGA INCREMENT.....DELCM = 2.00000000E+01 RADIANS/SEC.
CMEGA FINAL.....CMFIN = 1.05000000E+03 RADIANS/SEC.
V INITIAL.....VINIT = 0. IN./SEC.
V INCREMENT.....DELV = 1.00000000E+04 IN./SEC.
V FINAL.....VFIN = 2.00000000E+05 IN./SEC.

REFERENCES

1. Turner, M. Jonathan: Optimization of Structures to Satisfy Flutter Requirements. AIAA J., vol. 7, no. 5, May 1969, pp. 945-951.
2. Schmit, Lucien A., Jr.; and Thornton, William A.: Synthesis of an Airfoil at Supersonic Mach Number. NASA CR-144, 1965.
3. Ashley, H.; and McIntosh, S. C., Jr.: Application of Aeroelastic Constraints in Structural Optimization. Applied Mechanics, M. Hetényi and W. G. Vincenti, eds., Springer-Verlag, 1969, pp. 100-113.
4. Ashley, Holt: Aeroelasticity. Appl. Mech. Rev., vol. 23, no. 2, Feb. 1970, pp. 119-129.
5. Rudisill, Carl S.; and Bhatia, Kumar G.: Optimization of Complex Structures to Satisfy Flutter Requirements. AIAA J., vol. 9, no. 8, Aug. 1971, pp. 1487-1491.
6. Sheu, C. Y.; and Prager, W.: Recent Developments in Optimal Structural Design. Appl. Mech. Rev., vol. 21, no. 10, Oct. 1968, pp. 985-992.
7. Morrow, William M., II; and Schmit, Lucien A., Jr.: Structural Synthesis of a Stiffened Cylinder. NASA CR-1217, 1968.
8. Thornton, William A.; and Schmit, Lucien A., Jr.: The Structural Synthesis of an Ablating Thermostructural Panel. NASA CR-1215, 1968.
9. Fox, R. L.; and Kapoor, M. P.: Structural Optimization in the Dynamics Response Regime: A Computational Approach. AIAA J., vol. 8, no. 10, Oct. 1970, pp. 1798-1804.
10. Giles, Gary L.: Procedure for Automating Aircraft Wing Structural Design. J. Struct. Div., Amer. Soc. Civil Eng., vol. 97, no. ST1, Jan. 1971, pp. 99-113.
11. Stroud, W. Jefferson: Minimum-Mass Isotropic Shells of Revolution Subjected to Uniform Pressure and Axial Load. NASA TN D-6121, 1971.
12. Kavlie, Dag; and Moe, Johannes: Automated Design of Frame Structures. J. Struct. Div., Amer. Soc. Civil Eng., vol. 97, no. ST1, Jan. 1971, pp. 33-62.
13. Comm. on Metric Pract.: ASTM Metric Practice Guide. NBS Handbook 102, U.S. Dep. Com., Mar. 10, 1967.
14. Stein, Manuel; and Sanders, J. Lyell, Jr.: A Method for Deflection Analysis of Thin Low Aspect Ratio Wings. NACA TN 3640, 1956.
15. Houbolt, John Cornelius: A Study of Several Aerothermoelastic Problems of Aircraft Structures in High-Speed Flight. Prom. Nr. 2760, Swiss Fed. Inst. Technol. (Zürich), 1958.

16. Morgan, Homer G.; Huckel, Vera; and Runyan, Harry L.: Procedure for Calculating Flutter at High Supersonic Speed Including Camber Deflections, and Comparison With Experimental Results. NACA TN 4335, 1958.
17. Fiacco, Anthony V.; and McCormick, Garth P.: Nonlinear Programming: Sequential Unconstrained Minimization Techniques. John Wiley & Sons, Inc., c.1968.
18. Bracken, Jerome; and McCormick, Garth P.: Selected Applications of Nonlinear Programming. John Wiley & Sons, Inc., c.1968.
19. Moe, Johannes: Design of Ship Structures by Means of Non-Linear Programming Techniques. Symposium on Structural Optimization, AGARD CP No. 36, Oct. 1970, pp. 4-1 - 4-29.
20. Fox, Richard L.: Optimization Methods for Engineering Design. Addison-Wesley Pub. Co., Inc., c.1971.
21. Pierre, Donald A.: Optimization Theory With Applications. John Wiley & Sons, Inc., c.1969.
22. Wilde, Douglass J.; and Beightler, Charles S.: Foundations of Optimization. Prentice-Hall, Inc., c.1967.
23. Pope, G. G.; and Schmit, L. A., eds.: Structural Design Applications of Mathematical Programming Techniques. AGARDograph 149, Feb. 1971.
24. Anon.: Control Data 6000 Computer Systems Matrix Algebra Subroutines Reference Manual. Publ. No. 60135200, Control Data Corp., June 1966.
25. Hague, Donald S.; and Glatt, Curtis R.: An Introduction to Multivariable Search Techniques for Parameter Optimization (and Program AESOP). Contract No. NAS 2-4507, Boeing Co., Apr. 1968. (Available as NASA CR-73200.)
26. Clary, Robert R.: Practical Analysis of Plate Vibrations Using NASTRAN. NASTRAN: Users' Experiences, NASA TM X-2378, 1971, pp. 325-341.

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